Radiative transfer modeling of dust-coated Pancam calibration target materials: Laboratory visible/near-infrared spectrogoniometry

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Received 19 November 2005; revised 7 April 2006; accepted 15 May 2006; published 18 October 2006.

[1] Laboratory visible/near-infrared multispectral observations of Mars Exploration Rover Pancam calibration target materials coated with different thicknesses of Mars spectral analog dust were acquired under variable illumination geometries using the Bloomsburg University Goniometer. The data were fit with a two-layer radiative transfer model that combines a Hapke formulation for the dust with measured values of the substrate interpolated using a He-Torrance approach. We first determined the single-scattering albedo, phase function, opposition effect width, and amplitude for the dust using the entire data set (six coating thicknesses, three substrates, four wavelengths, and phase angles 3°–117°). The dust exhibited single-scattering albedo values similar to other Mars analog soils and to Mars Pathfinder dust and a dominantly forward scattering behavior whose scattering lobe became narrower at longer wavelengths. Opacity values for each dust thickness corresponded well to those predicted from the particles sizes of the Mars analog dust. We then restricted the number of substrates, dust thicknesses, and incidence angles input to the model. The results suggest that the dust properties are best characterized when using substrates whose reflectances are brighter and darker than those of the deposited dust and data that span a wide range of dust thicknesses. The model also determined the dust photometric properties relatively well despite limitations placed on the range of incidence angles. The model presented here will help determine the photometric properties of dust deposited on the MER rovers and to track the multiple episodes of dust deposition and erosion that have occurred at both landing sites.


1. Introduction

[2] The dynamic interaction between the Martian surface and atmosphere results in an active cycle of aeolian dust deposition and erosion. In visible and infrared wavelengths thin dust deposits (<100 μm) can obscure the spectral signature of underlying materials [e.g., Roush, 1982; Singer and Roush, 1983; Wells et al., 1984; Fischer and Pieters, 1993; Graff et al., 2001; Johnson et al., 2002] and complicate interpretations of surface composition from both in situ and remotely-sensed observations [e.g., Arvidson et al., 1989a, 1989b; Mustard and Sunshine, 1995; Crisp, 1998; McSween et al., 1999; Farrand et al., 2006; Bell et al., 2000, 2006; Christensen et al., 2001; Ruff et al., 2006]. Dust accumulation on the Mars Exploration Rover (MER) solar cells influenced available power during the missions [e.g., Arvidson et al., 2004a, 2004b], and dust coatings on the Pancam radiometric calibration targets (RCTs) complicated their use for reduction of multispectral reflectance data [e.g., Bell et al., 2004; J. F. Bell III et al., Multispectral analyses of fine-grained materials at the Mars Exploration Rover Spirit landing site in Gusev Crater, submitted to Journal of Geophysical Research, 2006, hereinafter referred to as Bell et al., submitted manuscript, 2006]. Similar effects were encountered during the Viking Landers and Mars Pathfinder missions [e.g., Arvidson et al., 1989b; Landis and Jenkins, 2000; Johnson et al., 2003].
Prior to the launch of the MER missions, laboratory observations of the Pancam RCTs were acquired in all Pancam wavelengths under many illumination geometries to characterize their spectrophotometric properties and improve Pancam radiometric and color calibration during mission operations [Bell et al., 2003, 2004, submitted manuscript, 2006]. In preparation for observations of the inevitably dust-coated RCTs on Mars we acquired additional laboratory spectrogoniometric measurements of the Pancam RCT materials coated with variable thicknesses of a Mars analog dust. In this paper, we use these data to test a new hybrid two-layer scattering model that combines a Hapke model [Hapke, 1993] for the dust coating with measured values of the substrate, interpolated using a He-Torrance model [He et al., 1991; Bell et al., 2003]. Our goal is to determine the degree to which dust spectrophotometric properties can be determined as a function of dust coating thickness and goniometric coverage when the (uncoated) substrate photometric properties are well known. This technique could be used with Pancam calibration target observations from Mars to better constrain the compositional and mineralogical properties of the Martian deposited dust, improve calibration accuracy of the Pancam images, and help with attempts to remove the spectral signature of dust coatings from observations of rocks and soils on Mars.

2. Methodology

2.1. Data Acquisition

The MER Pancam RCT materials are room temperature vulcanizing (RTV) surfaces with reflectances of approximately 20% (“black”), 40% (“gray”), and 60% (“white”). The Bloomsburg University Goniometer (BUG) [Shepard, 2001, 2002] was used to acquire bidirectional reflectances at four wavelengths similar to those provided by Pancam filters: 480 nm (Pancam L6 filter), 600 nm (L4), 750 nm (L2), 930 nm (R6). Band passes for the filters ranged from 16 nm to 30 nm. The BUG light source is a 100 W quartz-tungsten-halogen bulb. The output light is chopped, filtered, and focused onto a fiber optic bundle, which ends at the top of a goniometer arm with a 1.6 cm diameter lens assembly. The collimated output is directed onto the sample ~60 cm below. Samples are limited in size to <6.0 cm diameter. A calibrated silicon detector is mounted at the end of a second, longer goniometer arm (~90 cm). To reduce noise, the detector is electronically “locked” to the chopper motor on the source. Three software-controlled stepper motor stages are used to position the light source and detector in incidence, emission, and azimuth via a preprogrammed sequence of motions. The azimuth angle can be varied from 0° to 180°, the emission angle can be varied from 10° to 90° (measured from the horizontal), and the incidence angle can be varied from 15° to 90°. All measurements were calibrated using the reflectance standard Spectralon™ observed under near-normal incidence. The measurements were then converted into units of radiance factor as defined by Hapke [1993] via multiplication by cos(incidence)/cos(emission). Finally, a correction for stray light (induced by ambient room light) was applied. Measurement errors were estimated to be 5% for 480 nm and 600 nm data, and 10% for 750 nm and 930 nm data.

We used the dust deposition chamber constructed by Graff et al. [2001] to deposit uniform coatings of the Mauna Kea palagonitic soil HWMK919 [Morris et al., 2001] sieved to <53 μm grain size onto the calibration target materials. The dust was mechanically agitated inside a chamber assembly that consisted of a dust-tight enclosure, a mechanical agitation and circulation mechanism, and an electronic control box, all enclosed in an airtight case [Graff, 2003]. The mechanical agitation mechanism consisted of two cylindrical containers stacked vertically. The lower container housed a driving motor whose shaft was attached to two thin aluminum blades that performed the agitation. The lid to the upper container was perforated with ~2 mm holes to allow the dust generated to enter the chamber, while restricting larger particle aggregates from escaping. Dust coatings were deposited by placing ~20 g of
the HWMK919 dust into the upper container of the mechanical agitation mechanism. All samples to be coated were placed at the opposite end of the chamber. The dust deposition technique preferentially mobilized the finest fraction such that the average deposited grain diameter was about 10 ± 5 µm.

6 An aluminum disk was used as a witness plate to monitor dust thickness. A cover ring was placed on top of the aluminum disk to mask the outer edge and allow dust to be deposited only on the central portion. The optical thickness was determined using a vertically calibrated petrographic microscope, in which the change in focal length between the masked portion (cover ring removed) of the aluminum plate and average upper surface of the coating represented the coating thickness. Thickness measurements were taken in five locations across the plate and averaged. Mass measurements of the aluminum disk before and after dust deposition were also made to determine the net accumulated mass of deposited dust over the known surface area, which provided the thickness in units of mg/cm². Comparison of the optical thickness and mass measurements resulted in an estimated deposit density of <0.15 g/cm².

Six dust coating runs were conducted and resulted in mean dust thicknesses of 5 µm, 10 µm, 24 µm, 45 µm, 132 µm, and 225 µm. Uncertainties on the measured thicknesses were ≤ 5 µm. Representative images of the coatings on a solar cell (deposited during the same runs that coated the calibration target materials) are shown in Figure 1.

7 The Pancam views the calibration target on the rovers at an angle measured from the horizontal of about 38°. Times of day for calibration target acquisitions vary substantially on both rovers, although the majority has been acquired between 10 AM and 4 PM local time [cf. Bell et al., 2006]. Thus all BUG measurements were made with a fixed emission elevation of 38°, in order to match the Pancam viewing geometry. The measured incident geometries covered 25°–90° (measured from the horizontal), whereas azimuthal coverage was from 0° to 170° (Figure 2), resulting in phase angle coverage from 3° to 117°. The available piece of white RCT material was rectangular in shape and narrower than the BUG’s low incidence angle illuminated spot along one axis. As such, white substrate data could only be acquired in the principal plane and perpendicular to the principal plane, with the sample being physically rotated between collection runs. This resulted in collection of fewer data points for the white substrate, as only the 0° and 90° azimuth angles were measured (Figure 2). The directional hemispheric reflectances of the RCT substrates measured in the laboratory are shown in Figure 3 [cf. Bell et al., 2003] compared to the BUG data for the bare substrates integrated over all incidence angles. The deviation of the white substrate BUG values relative to the directional hemispherical data likely results from the sparse azimuthal coverage acquired and/or problems with the data resulting from the nonideal, rectangular shape of the white RCT material.

2.2. Two-Layer Model

Our previous adaptations of the Hapke bidirectional reflectance model for a two-layer medium [Hapke, 1993, p. 251] were used to study the spectrophotometric properties of JSC-1 Mars spectral analog dust [Allen et al., 1998] deposited onto rocks in the laboratory [Johnson and Grundy, 2001; Johnson et al., 2004] and dust deposition on the Mars Pathfinder calibration targets [Johnson et al., 2003]. Those models were numerically inverted using an iterative downhill simplex scheme [e.g., Nelder and Mead, 1965] to fit the coating thickness, the single-scattering albedo ω and the phase function P(Ω). To improve computational efficiency, those models ignored the opposition effect, specular reflections, and macroscopic roughness.
The model presented here differs from previous versions as follows. Rather than attempting to solve for
the Hapke parameters describing the lower substrate (as in the case of dust-coated rock studies), it allows the lower
layer to be replaced with an arbitrary substrate defined only by its Bidirectional Reflectance Distribution Function
(BRDF). The BRDF used in the current implementation is derived from models of laboratory measurements of the
Pancam calibration target materials [Bell et al., 2003]. The freedom of definition for the substrate material allows for
the accurate modeling of dust accumulation on materials that exhibit a strong specular scattering lobe (such as the
RTV silicone rubber used in the Pancam calibration targets [cf. Sohl-Dickstein et al., 2005]). Such materials are difficult
to describe using the traditional Hapke particulate model because the model includes no parameterization for a
specularly reflected component [e.g., Shepard et al., 1993].

Our adaptation of the Hapke two-layer model (described in detail in Appendix A) consists of substituting a
modified source function and diffuse contribution for \( \tau \geq \tau_0 \) into a rephrased integral for radiance at the detector.

\[
I_{D}(\Omega) = \frac{1}{\mu} \int_{\Omega} [F(\tau, \Omega) + D(\tau)] e^{-\mu L} d\tau
\]

where \( I_{D} \) is the radiance at the detector [Hapke, 1993, equation 9.26], \( F \) is the source function [Hapke, 1993,
9.27a], \( D \) is the diffuse component to the radiance at the detector, \( \mu_0 \) is \( \cos(\text{incidence}) \), \( \mu \) is \( \cos(\text{emission}) \), \( \tau \) is the
typical thickness, \( \tau_0 \) is the optical thickness at the interface between upper and lower layers, \( \Omega^0 \) and \( \Omega \) are the incident
and emission vectors respectively, \( \omega_\Omega \) is the single-scattering albedo of grains in the upper layer, \( r_L \) is the spherical
reflectance of the lower layer, \( p \) is the volume angular-scattering function of the upper layer, \( g \) is the phase
angle, \( q \) is the surface bidirectional scattering function of the lower layer, \( \phi(\tau) \) is the directionally averaged radiance, \( J_2 \) is the
portion of the directionally averaged radiance moving downward, and \( J \) is the irradiance incident upon the sample.

The result of these substitutions is to reduce the behavior of the lower layer to that of an infinitely thin
opaque membrane at \( \tau = \tau_0 \). The surface bidirectional scattering function \( q \) and reflectance \( r_L \) should be viewed as the analogues on this membrane of the single-scattering
albedo \( \omega_L \) and the volume angular-scattering function \( p \) in the upper layer. In addition, \( q \) is defined as the normalized
BRDF of the lower layer

\[
q(\Omega^0, \Omega) = \frac{BRDF_L}{BRDF_L(\Omega^0, \Omega)}
\]

The specific substrate model used in our study was that developed by the Pancam team to describe the Pancam
RCT [Bell et al., 2003]. This model consists of interpolation between BUG measured data points using a He-Torrance
model [He et al., 1991], a physical optics model borrowed from the realm of computer science, combined with a
Hapke backscatter term [Hapke, 1993]. The upper dust layer is modeled using the Hapke formalization, and the
sum of the lower and upper layer modeled radiances forms the radiance received at the detector. The Appendix
describes in detail the derivation of the equations used in the model.

The model accepts as input the BRDF values from the BUG measurements, which can be the entire data set or
subsets of data split by coating thicknesses, substrates, or specific geometries. The model simultaneously fits the
albedos for the lower substrate \((w_l)\) and the upper dust layer \((w_u)\), the dust layer optical thickness \(\tau\) over all
wavelengths, and the two-term Henyey-Greenstein (HG) phase function of the dust layer, which has the form:

\[
P(g) = \frac{c'(1 - b \delta^2)}{[1 + 2b \cos(\gamma) + b^2]^{3/2}} + \frac{(1 - c')(1 - b^2)}{[1 - 2b \cos(\gamma) + b^2]^{3/2}}
\]

where \( b \) is the asymmetry parameter and \( c' \) is the forward scattering fraction [cf. Johnson et al., 2006; J. R. Johnson et
al., Spectrophotometric properties of materials observed by Pancam on the Mars Exploration Rovers: 2. Opportunity,
submitted to Journal of Geophysical Research, 2006]. The opposition effect width \( (h) \) and magnitude \( (B0) \) are included in
the model and constrained to positive values \( \leq 1.0 \).

We note that other workers define the two-term HG function somewhat differently such that their \( c \) parameter
represents the backward scattering fraction [e.g., Hartmann and Domingue, 1998; Cord et al., 2003]. To be consistent
with those studies, we convert our forward fraction parameter \( c' \) to a backward fraction \( c' \) via \( c = (1 - c') \). A different
version of the two-term HG function was used by Hapke [1993, equation 6.18a] and McGuire and Hapke [1995]
in which their “\( c \)” parameter is related to \( c \) by the relation \( “c” = (2c - 1) \). McGuire and Hapke [1995] found that
artificial particles with various degrees of heterogeneity exhibited distinct \( b \) and \( c \) values related to deviations from a
particle’s spherical and internal perfection. Particles with microcracks, inclusions, or greater roughness exhibit low
and broad scattering lobes (small \( b \) values) and more pronounced backscattering (large \( c \) values). Smooth, clear spheres exhibit
larger asymmetry in their scattering lobes (large \( b \) values) but greater forward scattering (small \( c \) values). We will use their
results for comparison to the \( b \) and \( c \) values derived from the two-term HG models below.

The dust optical thickness \( \tau \) is defined as \( \tau = \int_{0}^{\infty} Ndz \) where \( N \) is the number density of particles in the coating
layer, \( \sigma \) is their cross-sectional area, and \( z \) is the altitude above the surface of the substrate. An approximate
relationship between \( \tau \), thickness \( (t) \), grain size, and porosity useful in comparing different models was derived by
Johnson et al. [2003]:

\[
\tau = -3*\tau*\ln(p)/2D
\]

where \( p \) is the fractional pore space (porosity) in the coating (equal to 1 minus the filling factor), and \( D \) is the grain size.
Models were run using a Levenberg-Marquardt least squares minimization routine with numerically calculated derivatives, and fits were evaluated using the reduced chi-square value ($\chi^2_r$) to compare how well the model was able to replicate coated substrate reflectances [e.g., Bevington and Robinson, 1992]. We note that $\chi^2_r$ values are sensitive to the number of data points used, estimated uncertainties in the data, and/or the magnitude of angular coverage chosen for a given data subset, such that direct comparison of $\chi^2_r$ values across disparate model types is inadvisable. Nonetheless, comparison of $\chi^2_r$ values within model families (described below) provides a useful indication of overall model sensitivity. If a given model and the assumed uncertainties were correct, $\chi^2_r$ values of 1.0 would be expected. In practice, however, reasonable agreement between models and observed spectra is found when $\chi^2_r$ is less than about 10.

### Results

Preliminary runs of the model demonstrated that normalization of the substrate surface bidirectional scattering function $q$ was problematic. The calibration target substrate BRDF was unconstrained for geometries outside of those measured using the BUG, and the He-Torrance model did not make sensible extrapolations for a subset of those unmeasured geometries. As a result, blind numerical normalization did not produce sensibly scaled scattering functions. Therefore we restricted the numerical integration of the substrate to geometries for which BUG data were acquired: Elevation was restricted to values between 30° and 50° for the emission vector, and between 20° and 90° for the incidence vector.

### Nominal Model Runs

The bidirectional two-layer reflectance model was fit first to the entire data set (all substrates, wavelengths, and dust thicknesses, comprising 10,860 measurements). In practice, most available Martian surface observations are limited in their viewing and illumination geometries (i.e., sampling less than a full BRDF). To simulate these limitations, subsequent model runs were made using subsets of the laboratory data constrained by geometry, substrate type, and dust thickness (Table 1) to provide better insight into the robustness of our methodology and its usefulness for interpretation of dust photometric behavior from observations acquired on Mars.

### Table 1. Matrix of Two-Layer Model Runs$^a$

<table>
<thead>
<tr>
<th>Substrates</th>
<th>Thicknesses</th>
<th>Geometries</th>
<th>Number of Points</th>
<th>$\chi^2_r$</th>
<th>$h$</th>
<th>$B_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W,G,B$</td>
<td>All</td>
<td>All</td>
<td>10860</td>
<td>5.65</td>
<td>0.105</td>
<td>1.00</td>
</tr>
<tr>
<td>$W,G,B$</td>
<td>All</td>
<td>No specular $W$</td>
<td>10192</td>
<td>2.90</td>
<td>0.067</td>
<td>1.00</td>
</tr>
</tbody>
</table>

### Substrate constraints

| $G$ | All | All | 4704 | 1.15 | 0.109 | 1.00 |
| $B$ | All | All | 4704 | 3.74 | 0.072 | 1.00 |

### Thickness constraints: Single

| $W,G,B$ | 5 $\mu$m | No specular $W$ | 1540 | 6.63 | 0.000 | 0.16 |
| $W,G,B$ | 10 $\mu$m | No specular $W$ | 1540 | 0.88 | 0.048 | 1.00 |
| $W,G,B$ | 24 $\mu$m | No specular $W$ | 1556 | 0.50 | 0.048 | 1.00 |
| $W,G,B$ | 45 $\mu$m | No specular $W$ | 1556 | 37.47 | 0.000 | 0.05 |
| $W,G,B$ | 132 $\mu$m | No specular $W$ | 1556 | 0.14 | 0.048 | 1.00 |
| $W,G,B$ | 225 $\mu$m | No specular $W$ | 1556 | 0.24 | 0.065 | 1.00 |

### Thickness constraints: Double

| $W,G,B$ | 225, 5 $\mu$m | No specular $W$ | 3096 | 1.18 | 0.050 | 1.00 |
| $W,G,B$ | 225, 10 $\mu$m | No specular $W$ | 3096 | 0.86 | 0.047 | 1.00 |
| $W,G,B$ | 225, 24 $\mu$m | No specular $W$ | 3096 | 0.47 | 0.064 | 1.00 |
| $W,G,B$ | 225, 45 $\mu$m | No specular $W$ | 3096 | 1.89 | 1.000 | 1.00 |
| $W,G,B$ | 225, 132 $\mu$m | No specular $W$ | 3112 | 1.01 | 1.000 | 1.00 |

### Geometric constraints

| $W,G,B$ | All | $i = 30–45^\circ$ No specular $W$ | 5987 | 2.36 | 0.056 | 1.00 |
| $W,G,B$ | All | $i = 30–90^\circ$ No specular $W$ | 9991 | 2.74 | 0.069 | 1.00 |
| $W,G,B$ | All | $i = 45–60^\circ$ No specular $W$ | 2184 | 2.92 | 0.096 | 1.00 |
| $W,G,B$ | All | $i = 45–90^\circ$ No specular $W$ | 4424 | 2.98 | 0.099 | 1.00 |
| $W,G,B$ | All | $i = 5–30^\circ$ No specular $W$ | 1736 | 4.23 | 0.122 | 1.00 |
| $W,G,B$ | All | $i = 60–70^\circ$ No specular $W$ | 1596 | 2.99 | 1.000 | 1.00 |
| $W,G,B$ | All | $i = 60–90^\circ$ No specular $W$ | 2996 | 3.02 | 0.033 | 1.00 |
| $W,G,B$ | All | $i = 70–80^\circ$ No specular $W$ | 1427 | 2.95 | 0.000 | 0.21 |
| $W,G,B$ | All | $i = 70–90^\circ$ No specular $W$ | 1987 | 2.95 | 1.000 | 0.00 |
| $W,G,B$ | All | $i = 80–90^\circ$ No specular $W$ | 1148 | 3.05 | 0.000 | 0.39 |

$^a$Values for $w$ are shown in Figures 6, 9, 13, 18, and 23. Values for $b$ and $c$ are shown in Figures 11, 15, 20, and 25. See text for discussion. $W$, white; $G$, gray; $B$, black.
faces for which the model reflectances were consistently lower than the observed reflectances. At $\sim 45 \mu m$ dust thickness, nearly equivalent contributions to the observed reflectance are made from both the dust and substrate layers, as described by Johnson et al. [2003, 2004]. Additional thickness measurements between the 45 $\mu m$ and 132 $\mu m$ data sets would provide the model with greater information to better constrain the transition from relatively thin to thick coatings. [21] The single-scattering albedo ($\omega$) spectrum of the dust derived from the model is shown in Figure 6 compared to $\omega$

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure4a}
\caption{Measured versus modeled bidirectional reflectances for the black, gray, and white RCT substrates derived from a two-layer model in which entire BUG data set was used. Line represents perfect correlation between measured and modeled data.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure4b}
\caption{Measured versus modeled bidirectional reflectances for the black, gray, and white RCT substrates derived from a two-layer model in which white substrate measurements acquired within 20° of specular geometries were excluded.}
\end{figure}
spectra derived for laboratory-deposited JSC-1 dust on rock substrates from Johnson et al. [2004] and to values modeled for dust deposited on the Mars Pathfinder calibration targets from Johnson et al. [2003]. Although the HWMK919 dust values are slightly higher than those from previous work, these differences likely are related to a combination of variations in grain shape, composition, internal imperfections, and degree of particle-to-particle contact (e.g., clumping) in the dust deposits, all of which affect the diffraction, transmission, and surface and volume scattering properties of dust grains [e.g., Pollack et al., 1979; van de Hulst, 1981; Petrova, 1993; Ockert-Bell et al., 1997; Kalashnikova and Sokolik, 2002; Johnson et al., 2003].

[22] The two-term HG scattering functions for the HWMK919 dust are shown in Figure 7 for each wavelength. Also shown is a hatched bar denoting the phase

Figure 5. Representative measured (solid symbols) and modeled (open symbols) relative reflectance spectra for different coating thicknesses (shown in top left of each panel) and substrates derived from a two-layer model in which entire BUG data set was used except for the specular geometries for the white substrate. Data were acquired at 60° incidence angle and averaged over all azimuths; error bars represent standard deviations of averaged values.

Figure 6. Single-scattering albedo values of HWMK919 dust derived from a two-layer model in which the entire BUG data set was used except for the specular geometries for the white substrate, compared to values derived from models of laboratory deposited JSC-1 dust on rock targets [Johnson et al., 2004] and models of air-fall-deposited dust on the Mars Pathfinder (MPF) calibration targets [Johnson et al., 2003].

Figure 7. Two-term Henyey-Greenstein single-scattering phase functions for the HWMK919 dust derived from the two-layer model in which the entire BUG data set was used except for the specular geometries for the white substrate. Hatched bar represents phase angles over which BUG data were acquired. The forward scattering behavior of the dust is evident, as is its wavelength-dependent nature (see text).
angle range of the BUG data. The dust is dominantly forward scattering, with the near-infrared exhibiting more forward scattering (less backscattering) than the visible wavelengths. This wavelength dependency could result from the effects of particle shape on scattering as well as the spectral reflectance properties of the HWMK919 dust. Shorter wavelength photons are more sensitive to features of an irregular particle’s shape at smaller spatial scales, and are thus more readily backscattered than photons at longer wavelengths. Additionally, the optical constants of palagonites within HWMK919 dust produce stronger absorption of shorter wavelength photons, so they are less likely than longer wavelength photons to pass through a particle unab- sorbed. Both effects result in more isotropic phase functions and increased surface scattering at shorter wavelengths.

[23] The opposition effect width \((h)\) and amplitude \((B0)\) were modeled as 0.067 and 1.0, respectively (Table 1). These values are not very well constrained, given the weak backscattering lobe of the dust (Figure 7). Nonetheless, the high \(B0\) value suggests that most light is scattered at the surface and the dust particles are rather opaque [e.g., Domingue et al., 1997]. The relatively small \(h\) value implies that the dust deposits are relatively porous or are consistent with lower ratios of largest to smallest particle size distribution (depending on the assumed particle size power law distribution) [cf. Helfenstein and Veverka, 1987; Hapke 1993]. This is consistent with the low measured densities of the dust deposits and their relatively uniform grain size distribution.

[24] Because the optical depth of the dust coating \((\tau)\) at normal (zenith) illumination is one of the fundamental properties derived by the two-layer model, it is useful to determine how \(\tau\) varies with measured dust coating thickness (Figure 8). By assuming a grain density of 3.0 g/cm\(^3\) for the HWMK919 particles and the maximum dust deposit density of 0.15 g/cm\(^3\), we calculated an approximate porosity of 0.95 for the dust deposit. Using this value in equation (5) along with different particle sizes demonstrates that the observed opacities of the dust deposits fall within an envelope defined by particle sizes between 5 \(\mu\)m and 15 \(\mu\)m. This is consistent with the estimated grain size distribution of the deposited HWMK919 dust.

### 3.2. Substrate Constraints

[25] We next investigated the model’s sensitivity to the type of substrate by only fitting BUG data from a single substrate (white, gray, or black). The necessity of removing the near-specular points in the white substrate data set restricts their phase angle coverage to \(22^\circ–82^\circ\), which is insufficient to constrain the photometric parameters very well. Figure 9 shows the dust single-scattering albedos derived from the black-only and gray-only models compared to the \(\omega\) values derived from the entire data set (excluding white substrate specular geometries). In the 480 nm and 600 nm wavelengths, the \(\omega\) values derived from models using only the gray substrate data were lowest, whereas those derived from black substrate models were the highest. To investigate this difference, it is useful to compare the two-term HG single-scattering phase functions for each wavelength derived from each model, as shown in Figure 10. The dust phase functions at 600, 750, and 930 nm derived from the black substrate models appear to be the most forward scattering, whereas the dust from the gray substrate models suggest the least forward scattering. The similarity of all 480 nm phase functions is partly a consequence of the low reflectance of the HWMK919 dust at 480 nm. Because 480 nm is the only wavelength where the reflectance of the dust is lower than all of the substrate types, reflectances from each type of coated substrate decrease monotonically with increasing dust thickness. This results in highly consistent phase functions among all substrate types. However, because our observations do not measure the forward scattering at phase angles greater than \(117^\circ\) (denoted by the absence of hatched bars in Figure 10) the model is underconstrained at phases >\(117^\circ\), and the differences in Figure 10 may not be necessarily accurate. For example, a lower observed radiance from the dusted surface...
may either be compensated in the model by placing more flux into the “unobserved” forward lobe (g > 117°) or by making the single-scattering albedo lower (or some combination of the two). Unfortunately, the lack of data at g > 117° allows subtle inflections in the shape of the two-term HG curve below 117° to heavily influence the shape at g > 117°, with the single-scattering albedo adjusting itself in step (perhaps inappropriately) to match the overall reflectance.

Figure 11 compares the b and c values of the synthetic particles studied by McGuire and Hapke [1995] to those derived from models using all data (with and without the specular data for the white substrate) and the gray and black single-substrate cases. For all models, the deposited dust particles behave most similarly to spheres with few internal scatterers but various degrees of surface roughness and/or to irregularly shaped particles. As was evident in Figure 10, greater forward scattering (smaller c values) and narrower scattering lobes (larger b values) occurred with increasing wavelength for all but the gray substrate model. The gray substrate model exhibited average two-term HG functions with slightly broader scattering lobes (smaller b values) than for the black substrate models. The observations suggest that a substrate with reflectance values similar to the dust deposit constrains the two-layer model less well than substrates with reflectances appreciably lower (or higher) than the dust.

Table 1 lists the $\chi^2_v$, h, and $B_0$ values associated with the single-substrate models. The larger $\chi^2_v$ value for the black substrate model doesn’t necessarily represent a poorer model fit, but may result from an underestimation of the errors for the black substrate data. Larger estimated errors would result in a smaller $\chi^2_v$ value [Bevington and Robinson, 1992]. The modeled average $B_0$ values were the same (1.00) for the single-substrate models as for the models using the full data set. The average h value for the black substrate model was relatively similar to that derived from the full data set (absent the specular geometries for the white substrate), whereas the h value derived from the gray substrate model was larger. This implies that the dust surface appears less porous to the gray substrate model, which results from the similarity between the gray substrate average reflectance and that of the dust. Compared to the dust and black substrate, the reduced spectral contrast between the dust and gray substrate hampered the model’s ability to discriminate between the two surfaces.

We conclude from these models that while observations of a single dust-coated substrate may provide useful information on the general nature of the dust phase function, multiple substrates (preferably covering a range of reflectances larger and smaller than that of the dust) provide a more robust description of the dust photometric properties.

### 3.3. Thickness Constraints

During the course of a mission multiple observations of a rover or lander deck on Mars will sample variable thicknesses of dust deposits. Given this likelihood, we next tested how restrictions on the number and/or thickness of available coatings would affect the overall model fits and derived dust parameters. We considered cases where only a single thickness of a coated substrate was used and cases where two thicknesses were used (225 µm and one other coated sample). In all cases the specular geometries for the white substrate were excluded from the model. Table 1 lists the $\chi^2_v$, h, and $B_0$ values for these models.
3.3.1. Single-Thickness Models

[30] For the single-thickness models, the \( \chi^2 \) values tended to decrease with increasing dust cover, except for the 45 \( \mu m \) thickness. This is illustrated in Figure 12, where representative data acquired at 60° incidence angle demonstrate that the 45 \( \mu m \) model spectra were most dissimilar from the measured spectra. As described above, at this thickness nearly equivalent contributions from both the dust and substrate hamper the model’s ability to discriminate between the two layers [Johnson et al., 2003, 2004]. Also, the 5 \( \mu m \) thickness models underestimated the measured values to a lesser extent, ostensibly due to the small radiometric contribution of the 5 \( \mu m \) dust layer. The \( h \) values of the single-thickness models ranged from 0.048 to 0.067 and \( B0 = 1.00 \) for all but the 5 \( \mu m \) and 45 \( \mu m \) thickness models (Table 1), whose \( h \) values (0.000) and \( B0 \) values (0.05–0.16) are likely spurious given their dissimilarity from other model results.

[31] The single-scattering albedos derived for the HWMK919 dust from each of the single-thickness models (Figure 13) are comparable to those using the entire data set (Figure 6) for only the 24 \( \mu m \), 132 \( \mu m \), and 225 \( \mu m \) model runs. The 5 \( \mu m \) and 45 \( \mu m \) modeled \( \omega \) values showed little variation with wavelength, consistent with their relatively poorer model fits. The 10 \( \mu m \) model run exhibited reasonable \( \omega \) values at all wavelengths except 930 nm. This suggests that photons at shorter wavelengths may be more readily absorbed or scattered by a 10 \( \mu m \) dust layer than the longest wavelength photons which can penetrate more readily to the substrate layer.

[32] The two-term HG functions for the single-thickness models are shown in Figure 14 and the \( b \) and \( c \) values are plotted in Figure 15. Consistent with the above discussion, the 5 \( \mu m \) and 45 \( \mu m \) models exhibited spurious results, as indicated by \( b \) and \( c \) values that plot far from the McGuire and Hapke [1995] L-shaped trend of artificial particle values (Figure 15). The 10 \( \mu m \) model results were most like the bare substrates (which contain a strongly forward scattering lobe [Sohl-Dickstein et al., 2005]), whereas the 132 \( \mu m \) and 225 \( \mu m \) model results revealed a slightly broader and less forward scattering lobe (most consistent with the irregular particles of McGuire and Hapke [1995]). Results for the 24 \( \mu m \) model fell between the 10 \( \mu m \) and thickest coatings.

[33] The modeled dust optical thickness (\( \tau \)) values for the dust derived from the single-thickness coatings are shown in Figure 16 compared to the estimated dust grain size calculated using equation (5). The \( \tau \) value for the 225 \( \mu m \) model was appreciably higher than previous results and suggests that at this depth the single-thickness model may underestimate the effective grain size of the deposited dust particles.

3.3.2. Double-Thickness Models

[34] Results for models in which the 225 \( \mu m \) data were paired with another deposit thickness revealed somewhat
better constrained results than the single-thickness models, and modeled $x^2$ values were overall more consistent. In particular, the 45 $\mu$m data were better constrained once paired with the 225 $\mu$m data, as shown in Figure 17 where representative data acquired at 60° incidence angle are compared to the model results. The (5 $\mu$m, 225 $\mu$m) thickness models slightly overestimated the measured values for the white substrate, although the $x^2$ value was substantially improved compared to the single-thickness model (Table 1). The $h$ values of the double-thickness models ranged from 0.034 to 0.064 except for the (45 $\mu$m, 225 $\mu$m) model’s value of 1.000. As discussed above, distinguishing between the two layers is difficult for the model when the 45 $\mu$m data are used. $B_0$ values were equal to 1.00 for all models.

[35] The single-scattering albedos derived for the HWMK919 dust from each of the double-thickness models (Figure 18) were similar to those using the entire data set (Figure 6). Two exceptions were the significantly lower $\omega$ values for the (45 $\mu$m, 225 $\mu$m) model and the slightly lower $\omega$ values for the (132 $\mu$m, 225 $\mu$m) model. This suggests that as long as samples with thin (e.g., <30 $\mu$m) and thick (e.g., >100 $\mu$m) coating thickness are paired in the two-layer model, the $\omega$ values derived from double-thickness models will be relatively consistent.

[36] The two-term HG functions for the HWMK919 dust derived from the double-thickness models are shown in Figure 19 and the $b$ and $c$ values are plotted in Figure 20. The models that used thin coatings (5 $\mu$m, 10 $\mu$m, 24 $\mu$m) paired with the 225 $\mu$m data exhibited more narrow forward scattering behaviors compared to the thicker coatings (45 $\mu$m, 132 $\mu$m). This is similar to the results of the single-thickness models (Figures 14 and 15), although the double-thickness models exhibited greater consistency, particularly for the (45 $\mu$m, 225 $\mu$m) model. The outlier point for the (132 $\mu$m, 225 $\mu$m) model in Figure 20 was from the 930 nm model and is considered to be a spurious result given that its asymmetry value of 0.00 implies a isotropic scattering function for the dust, which is inconsistent with other model results.

Figure 12. Representative measured (solid symbols) and modeled (open symbols) relative reflectance spectra for two-layer model runs in which only data from a single dust thickness were used (shown in top left of each panel). Data were acquired at 60° incidence angle and averaged over all azimuths; error bars represent standard deviations of averaged values.

Figure 13. Single-scattering albedos for the HWMK919 dust at each wavelength derived from single-thickness two-layer models (thickness shown in legend). Albedo spectra from 24 $\mu$m, 132 $\mu$m, and 225 $\mu$m models are reasonable, whereas 5 $\mu$m and 45 $\mu$m model results are not. The 10 $\mu$m model results are realistic for all wavelengths but 930 nm (see text).
The modeled \( \tau \) values for the dust derived from the double-thickness models were also well constrained (Figure 21) and fall within the envelope of estimated 5–15 \( \mu \text{m} \) grain size for the HWMK919 dust (see Figure 16).

3.4. Geometric Constraints

Repeated observations on Mars often cover limited illumination geometries owing to mission operational constraints. Therefore we next examined the effects of limiting the incidence angle coverage on the model’s ability to replicate measured data and model and HWMK919 dust
photometric parameters. Table 1 lists the incidence angle subsets used and their corresponding $\chi^2$ values, which varied only slightly among models despite significantly different incidence angle ranges. The highest $\chi^2$ values occurred for incidence angles close to horizontal [5°–30°] and near nadir [60°–90°, 80°–90°], whereas the lowest $\chi^2$ values were found for intermediate incidence angles [30°–45°]. This may result from better illumination of both the dust and substrate at intermediate incidence angles compared to grazing or nadir incidence angles. Alternatively, the greater number of data points and phase angles in the [30°–45°] data set (Table 1) may simply provide better leverage for the model fits. Similarly, modeled $h$ values were most consistent with previous model runs (i.e., within a range from ~0.040 to 0.100) when derived from models in which intermediate incidence angles were used. For comparison, $B_0$ values derived from these models were consistently equal to 1.00 except for angles near nadir, where a specular reflection might contribute to an apparent opposition surge.

[39] Figures 22a and 22b illustrate some examples of measured and modeled relative reflectance spectra derived from the geometrically constrained models for 24 μm and 132 μm thicknesses. Figures 22a (top) and 22b (top) provide representative spectra extracted for an incidence angle of 60° and averaged over all azimuths; error bars represent standard deviations of averaged values.

**Figure 17.** Representative measured (solid symbols) and modeled (open symbols) relative reflectance spectra for two-layer model runs in which only data from the 225 μm thickness and another dust thickness were used (shown in top left of each panel). Data were acquired at 60° incidence angle and averaged over all azimuths; error bars represent standard deviations of averaged values.

**Figure 18.** Single-scattering albedos for the HWMK919 dust at each wavelength derived from two-layer models in which only data from the 225 μm thickness and another dust thickness were used (thickness shown in legend). All albedo spectra are consistent with the exception of the (45 μm, 225 μm) model results.
angle of $30^\circ$, whereas the bottom panels use spectra extracted from $90^\circ$ (nadir) incidence. In all cases the modeled spectra replicated the measured spectra very well, although the incidence angle ranges clustered near nadir $[60^\circ, 90^\circ]$ exhibited slightly worse fits, consistent with their slightly elevated $\chi^2$ values. [40] This consistency among the geometrically constrained models is emphasized in Figure 23, which shows the similarity among all single-scattering albedo spectra derived from the models. The only discernible difference is a slightly elevated albedo at 480 nm for the $[80^\circ, 90^\circ]$ incidence angle model.

Figure 19. Two-term Henyey-Greenstein single-scattering phase functions for the HWMK919 dust for each wavelength derived from the two-layer models in which only data from the 225 $\mu$m thickness and another dust thickness were used (thickness shown in legend).

Figure 20. Asymmetry parameters ($b$) versus forward scattering fraction parameters ($c$) for artificial particles from McGuire and Hapke [1995] compared to results for HWMK919 dust derived from two-layer models in which only data from the 225 $\mu$m thickness and another dust thickness were used (thickness shown in legend).

Figure 21. Optical depths for each coating thickness derived from two-layer models in which only data from the 225 $\mu$m thickness and another dust thickness were used. Lines were computed using equation (5) and labeled grain size diameters, assuming a porosity of 0.95 (see Figure 8).
The two-term HG functions for the HWMK919 dust derived from these models are plotted in Figure 24 and the \( b \) and \( c \) values are shown in Figure 25. With the exception of the \([80^\circ - 90^\circ]\) model the derived scattering functions were quite similar, and all showed a consistent increase in forward scattering (smaller \( c \) values) and narrower scattering lobes (larger \( b \) values) with increasing wavelength, similar to the models that used the entire data set (see Figure 7).

The dust \( \tau \) values derived from these models also were relatively well constrained (Figure 26). All fell within the 5–15 \( \mu \)m grain size envelope for the HWMK919 dust except the \([5^\circ - 30^\circ]\) and \([80^\circ - 90^\circ]\) incidence angle ranges for the 132 \( \mu \)m thickness model. The models that used these two incidence angle sets resulted in lower grain size estimates than the other models at thicknesses greater than 45 \( \mu \)m.

The relative insensitivity of the parameters derived by the two-layer model to the range of input incidence angles provides confidence that the model is well behaved and that the nature of scattering from a dusted surface is more influenced by the thickness of the dust deposit and the reflectance of the substrate materials than the incident illuminations.

4. Discussion and Conclusions

The multispectral goniometric data presented here provided a useful data set with which to test a hybrid two-layer radiative transfer scattering model. By varying the type of Pancam RCT substrate, thicknesses of HWMK919 dust, and incidence angle ranges input to the model, we investigated the model’s ability to produce consistent and reasonable spectrophotometric properties for the dust. Our results suggest that the best characterization of the dust properties results from using substrates whose reflectances are either greater or less than the single-scattering albedos of the deposited dust. If the model uses observations of only a single dust thickness, the resulting dust properties are less well constrained than if data from both a relatively thick and thin dust coating are used. The models suggest that by 225 \( \mu \)m dust thickness the substrate properties are completely masked by the dust (Figures 5 and 12). The model is capable of determining the dust photometric properties relatively well even if the range of incidence angles is limited. Although we acquired data at only a single emission angle (for the purposes of investigating the model’s applicability to the Pancam calibration target materials on Mars), additional tests exploring the model’s sensitivity to various emission angle restrictions would be a useful supplementary study. In particular, this would be useful in subsequent studies that may use similar two-layer models to constrain the dust deposit optical thickness on lander/rover surfaces [cf. Johnson et al., 2003, 2004].

One of the unexpected results from this work is that bidirectional reflectance properties of the deposited dust...
were very forward scattering, unlike the more backscattering nature of typical Mars soils observed by the Viking Lander, Mars Pathfinder Lander, and Spirit Mars Exploration Rover [e.g., Arvidson et al., 1989a; Johnson et al., 1999, 2006]. In their two-layer models of JSC-1 dust deposited on rocks, Johnson et al. [2004] did not report one-term HG phase function parameter values, but upon review of those models it was confirmed that asymmetry parameters as high as +0.19 (i.e., forward scattering) were modeled for the deposited dust surfaces. The modeling of dust-coated Mars Pathfinder calibration targets by Johnson et al. [2003] could have provided another useful comparison, but those authors assumed the deposited dust to scatter isotropically to minimize the number of free parameters in their model.

[46] However, the forward scattering behavior of these deposited dusts is more consistent with that of atmospheric dust as modeled by Tomasko et al. [1999], Lemmon et al. [2004] and Lemmon and the Athena Science Team [2006] over the Mars Pathfinder and MER sites. Figure 27 shows the two-term HG functions derived from the two-layer model using all data (except the specular geometries for the white substrate; see Figures 7 and 10) compared to the phase functions derived for the atmospheric dust at the MER sites by Lemmon et al. [2004] and Lemmon and the Athena Science Team [2006]. The atmospheric dust phase functions represent components derived from reflectance and transmission, but with the diffraction component removed to more accurately simulate deposited dust grains. The similarity between the atmospheric and laboratory-

Figure 22b. Representative measured (solid symbols) and modeled (open symbols) relative reflectance spectra for the 132 µm thickness model runs in which data from a restricted range of incidence angles were used (shown in top left of each panel). (top) Data shown were acquired at 30° incidence angle (averaged over all azimuths); white substrate data were restricted to avoid specular geometries, and hence data are not shown. (bottom) Data acquired at 90° incidence angle (averaged over all azimuths). Error bars represent standard deviations of averaged values.

Figure 23. Single-scattering albedos for the HWMK919 dust at each wavelength derived from two-layer models in which data from a restricted range of incidence angles were used (shown in legend).
deposited dusts suggests that the effective grain size, porosity, and reflectance properties between the two dust types are relatively similar, although the dissimilarity between the 440 nm/480 nm phase functions requires further study.

Figure 24. Two-term Henyey-Greenstein single-scattering phase functions for the HWMK919 dust for each wavelength derived from the two-layer models in which data from a restricted range of incidence angles were used (shown in top left of each panel).

The anomalous data acquired for the white calibration target when viewed in specular geometries likely results from the surface texture of the material combined with its high reflectance. The materials were made from molds constructed by D. Britt (personal communication, 2006) that were intentionally pitted to result in small (~40 μm)

Figure 25. Asymmetry parameters (b) versus forward scattering fraction parameters (c) for artificial particles from McGuire and Hapke [1995] compared to results for HWMK919 dust derived from two-layer models in which data from a restricted range of incidence angles were used (shown in legend).

Figure 26. Optical depths for each coating thickness derived from two-layer models in which data from a restricted range of incidence angles were used (shown in legend). Lines were computed using equation (5) and labeled grain size diameters, assuming a porosity of 0.95 (see Figure 8).
bumps on the calibration target material surfaces. This was done to minimize the narrow specular lobe observed in previous versions of these materials, and the specular lobe became broader as a result of this process. All surfaces exhibit this lobe to some extent [cf. Bell et al., 2003], but the lower reflectance of the gray and black surface types minimized the broad specular lobe for those surfaces. A useful analog to the behavior of the white calibration target surface textures is the bright Lunar Lake (Nevada) playa surface studied by Shepard et al. [1993] that also exhibited broad specular scattering lobes in reflectance observations. Electron microscope images showed that the playa materials contained surfaces with millimeter-scale undulations that were coated with /C24/10/C22/ ellipsoidal particles. This likely resulted in the specular scattering lobes in a similar manner to the /~10 /mellipsoidal particles. This likely resulted in the specular scattering lobes in a similar manner to the /~40 /m bumps on the white calibration target material.

Future work will apply the two-layer model derived here to Pancam multispectral data acquired of the RCTs and solar cells on the decks of the MER rovers (cf. K. M. Kinch et al., A preliminary analysis of the dust deposition data from the Panoramic Camera (Pancam) calibration targets on the Mars Exploration Rovers, submitted to Journal of Geophysical Research, 2006). These surfaces have experienced multiple episodes of dust deposition and erosion resulting from variations in atmospheric dust content and wind activity at the two landing sites. As such, they will present a challenging test of the two-layer model’s ability to determine spectrophotometric properties of air-fall-deposited dust on Mars. In particular, the precise geometric microstructure and temporal stability of deposited dust particles on Mars is likely sufficiently dissimilar compared to laboratory-deposited dusts to warrant additional investigations into the effects of clumping and particle aggregation that might simulate the redistribution of originally air-fall-deposited particles onto lander/rover surfaces. Such work will be relevant to observations acquired by the SSI camera on the Phoenix lander [Smith, 2003] and the MastCam and MAHLI cameras onboard the Mars Science Lander [Malin et al., 2005; Edgett et al., 2005].

Appendix A

[48] The two-layer radiative transfer model used in this work is an adaptation of the Hapke [1993, section 9.D] two-layer formalization to cases where the lower layer is not amenable to representation using a standard Hapke particulate description. This adaptation allows the lower layer to be described solely by its bidirectional reflectance distribution function (BRDF). That is, the pertinent properties of the substrate are its behavior at the interface with the upper layer.

[50] In our application of this hybrid model the lower substrate was empirically measured [Bell et al., 2006], and
interpolation between measured data points was performed using a He-Torrance model [He et al., 1991]. The upper (dust coating) layer was described using the standard particulate Hapke model. The derivation of the modified Hapke two-layer model to allow arbitrary substrates proceeds as follows: [51] Begin with a modified version of equation 9.26 of Hapke [1993] describing the radiance at the detector \( I_d(\Omega) \) from an optically thin layer overlaying a thick substrate

\[
I_d(\Omega) = \int_0^{\tau_0} \frac{1}{\mu} [F(\tau, \Omega) + \omega_U \varphi(\tau)] e^{-\mu \tau} d\tau + \int_{\tau_0}^\infty \frac{1}{\mu} [F(\tau_0, \Omega) + r_L I_2(\tau_0)] e^{-\mu \tau} d\tau
\]  

\( \mu \) is cos(emission), \( F \) is the source function [Hapke, 1993, equation 9.27a], \( \tau \) is the optical thickness, \( \Omega \) is the emission vector, \( \omega_U \) is the single-scattering albedo of grains in the upper layer, and \( \varphi(\tau) \) is the directionally averaged radiance. In the two layer case the radiance at the detector can be split into the sum of radiances reaching the detector from the upper and lower layers. This sum is written as

\[
I_d(\Omega) = \int_0^{\tau_0} \frac{1}{\mu} [F(\tau, \Omega) + \omega_U \varphi(\tau)] e^{-\mu \tau} d\tau + \int_{\tau_0}^\infty \frac{1}{\mu} [F(\tau_0, \Omega) + r_L I_2(\tau_0)] e^{-\mu \tau} d\tau
\]  

\( \tau_0 \) is the optical thickness of the substrate, \( r_L \) is the albedo of the substrate, and \( I_2(\tau) \) is the downwelling diffuse radiance per solid angle at depth \( \tau \), i.e., \( I_2(\tau_0) \) is the diffuse radiation that reaches the lower substrate from the upper layer. Hence the quantity \( r_L \) times \( I_2(\tau_0) \) is the diffuse radiance that has been reflected back upward after interacting with the lower substrate. Following integration of the right-hand term, and noting the \( \tau \) independence of \( I_2 \) and \( F \) for \( \tau \geq \tau_0 \), this becomes

\[
I_d(\Omega) = \int_0^{\tau_0} \frac{1}{\mu} [F(\tau, \Omega) + \omega_U \varphi(\tau)] e^{-\mu \tau} d\tau + \int_{\tau_0}^\infty \frac{1}{\mu} [F(\tau_0, \Omega) + r_L I_2(\tau_0)] e^{-\mu \tau} d\tau
\]  

The source function \( F \) is expressed as

\[
F(\tau, \Omega) = J \frac{\omega_U}{4\pi} p_L(\tau, g) e^{-\mu_0 \omega_U} \quad \tau < \tau_0 \\
F(\tau, \Omega) = \mu_0 \omega_U r_L q(\Omega', \Omega) / \pi \quad \tau \geq \tau_0.
\]  

where \( J \) is the irradiance incident upon the sample, \( p_L(\tau, g) \) is the volume angular-scattering function of the upper layer, \( g \) is the phase angle, \( \mu_0 \) is cos/incidence, \( \Omega' \) and \( \Omega \) are the incident and emission vectors respectively, and \( q(\Omega', \Omega) \) is the surface bidirectional scattering function. The surface bidirectional scattering function can be thought of as either the surface analogue of the volume angular scattering function, or as the normalized BRDF of the substrate

\[
q(\Omega', \Omega) = \frac{\text{BRDF}_L}{(\text{BRDF}_L)_{\Omega', \Omega}}\quad (A4)
\]  

The formulation for \( F \) in the upper layer in (A4) follows equation 9.27a of Hapke [1993]. The formulation for the lower layer includes modifications inherent in describing the source function with respect to a surface rather than a volume. These consist of a factor of 2 stemming from normalization of the scattering function over a hemisphere instead of a sphere, a factor of \( \mu_0 \) resulting from the description of \( q \) without consideration for the cosine dependence of a surface scattering function on the incidence angle, and a second factor of 2 stemming from the normalization of \( \mu_0 \) over a hemisphere. In addition, \( r_L \) takes on the role of \( \omega_U \).

[53] To determine the diffuse upwelling \( I_1(\tau_0) \) and downwelling \( I_2(\tau) \) diffuse radiance components, we begin with equation 8.38 of Hapke [1993]:

\[
-\frac{1}{4} \frac{d^2 \varphi(\tau)}{d\tau^2} = -\gamma_U^2 + J \frac{\omega_U}{4\pi} e^{-\mu_0 \omega_U} \quad (A6a)
\]

\[
\frac{d\varphi(\tau)}{d\tau} = I_1(\tau) - I_2(\tau) \quad (A6b)
\]

where \( \gamma_U = (1 - \omega)^{1/2} \) is the albedo factor of the upper layer. The average diffuse radiance is computed as

\[
\varphi(\tau) = \frac{I_1(\tau) + I_2(\tau)}{2} \quad (A7)
\]

[54] Following Hapke’s [1993] equation 8.44, (A6b) and (A7) can be combined with the boundary condition of no downwelling diffuse radiance at the top of the upper layer

\[
I_2(\tau) = 0 \quad (A8)
\]

in order to produce the equation

\[
\varphi(0) = \frac{1}{2} \frac{d\varphi(0)}{d\tau} \quad (A9)
\]

[55] It further follows that the upwelling diffuse radiance immediately above the lower layer is equal to the downwelling diffuse radiance reaching the lower layer multiplied by the albedo of the lower substrate (photons which have already been scattered once in the upper layer) plus the directional flux impacting on the lower layer multiplied by the albedo of the lower layer (photons whose first scattering event occurs in the lower layer). This provides the further boundary condition

\[
I_1(\tau_0) = r_L I_2(\tau_0) + r_L J \frac{\mu_0}{2\pi} e^{-\mu_0 \omega_U} \quad (A10)
\]

Using equations (A6b) and (A7) this can be rewritten in terms of the directionally averaged radiance and its first derivative at \( \tau_0 \).

\[
\varphi(\tau_0) = \frac{r_L + 1}{2(\tau_L - 1)} \frac{d\varphi(\tau_0)}{d\tau} - \frac{r_L J \mu_0 e^{-\mu_0 \omega_U}}{2\pi(\tau_L - 1)} \quad (A11)
\]
The following diffuse radiance equations and their derivatives from equation 8.39 of Hapke [1993] apply for $0 \leq \tau \leq \tau_0$

\[ \varphi(\tau) = Ae^{-2\gamma_U \tau} + Be^{2\gamma_U \tau} + Ce^{-\gamma_U \tau} \]  
(A12a)

\[ \frac{d\varphi(\tau)}{d\tau} = -2\gamma_U Ae^{-2\gamma_U \tau} + 2\gamma_U Be^{2\gamma_U \tau} - \frac{1}{\mu_0} Ce^{-\gamma_U \tau} \]  
(A12b)

\[ \frac{d^2\varphi(\tau)}{d\tau^2} = 4\gamma_U^2 Ae^{-2\gamma_U \tau} + 4\gamma_U^2 Be^{2\gamma_U \tau} + \frac{1}{\mu_0} Ce^{-\gamma_U \tau} \]  
(A12c)

Through a series of algebraic manipulations, these can be solved using (A6a), (A9) and (A11). The constants $A$, $B$, and $C$ become

\[ A = \frac{(1 - \gamma_U)B}{(1 + \gamma_U)} - \frac{\left(1 + \frac{1}{2\mu_0}\right) C}{(1 + \gamma_U)} \]  
(A13a)

\[ B = \left[ \frac{\alpha_0 + 1}{2(\tau_1 - 1)} \right] \frac{2\gamma_U}{\left(1 + \gamma_U\right)} e^{-2\gamma_U \tau_0} - \frac{\alpha_0 + 1}{2(\tau_1 - 1)} \frac{1}{\mu_0} e^{-\gamma_U \tau_0} + \frac{1}{\tau_1} e^{-\gamma_U \tau_0} - \frac{1}{\mu_0} e^{-\gamma_U \tau_0} \]  
(A13b)

\[ C = \frac{J}{4\pi} \frac{4\omega_U \mu_0^2}{4\gamma_U \mu_0 - 1} \]  
(A13c)

With substitutions and integrations, these four components reduce to

\[ I_{D,\text{upper-direct}}(\Omega) = \frac{\mu_0}{\mu_0 + \mu} \frac{J\omega_U \mu_U(g)}{4\pi} \left[ 1 - e^{-\gamma_U \tau_0} \right] \]  
(A16a)

\[ I_{D,\text{upper-diffuse}}(\Omega) = \omega_U \left( \frac{A}{2\gamma_U \mu + 1} \left[ 1 - e^{-2\gamma_U \tau_0} \right] + \frac{B}{2\gamma_U \mu + 1} \left[ 1 - e^{-2\gamma_U \tau_0} \right] + \frac{C}{\mu_0 + 1} \left[ 1 - e^{-\gamma_U \tau_0} \right] \right) \]  
(A16b)

\[ I_{D,\text{lower-direct}}(\Omega) = J \frac{\mu_0 \omega_U g(\Omega', \Omega)}{\pi} e^{-\gamma_U \tau_0} \]  
(A16c)

\[ I_{D,\text{lower-diffuse}}(\Omega) = \frac{2\varphi(\tau_0)}{2} - \frac{d\varphi(\tau_0)}{d\tau} e^{-\gamma_U \tau_0} \]  
(A16d)

\[ \text{BRDF} = \frac{I_D}{J \mu_0} \]  
(A17)

Finally, the bidirectional reflectance distribution function (BRDF) is defined as

Note that this final formulation is independent of $J$ because $J$ occurs in $I_D$ (from its inclusion in equations (A16a), (A16c), and (A13c)). Equation (A17) provides the principal output of the two-layer model presented here.

Acknowledgments. This work was performed under the Planetary Geology and Geophysics Program (PG&G), contract W-10, 037. M. Shepard and the BUG Lab were supported by a grant from NASA PG&G. W. Grundy acknowledges support from NASA PG&G grant NNG04G172G to Lowell Observatory. Additional support was provided by the Mars Exploration Rover mission. Any use of trade or product names in this publication is for descriptive purposes only and does not imply endorsement by the U.S. government. We thank M. Lemonon for supplying the atmospheric dust phase function data and T. McCord and an anonymous reviewer for helpful comments.

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