Numerical Simulation of the Atmospheric Circulation and Climate of Mars

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ABSTRACT

The Mintz-Arakawa two-level model for planetary atmospheres has been adapted to simulate the atmospheric circulation and climate of Mars. The model uses the primitive equations of atmospheric motion, with heating and cooling by solar and infrared radiative transfer and by turbulent convection. Carbon dioxide is the principal atmospheric constituent and is allowed to condense on the planet’s surface, releasing latent heat where the surface cools to the CO₂ frost point. Two numerical experiments are made; one simulates orbital conditions at the southern summer (northern winter) solstice of Mars, and the other orbital conditions at the southern autumnal equinox.

The results of the solstice experiment show strong zonal mean west winds in the middle and high latitudes of the winter hemisphere produced by the net eastward Coriolis torque that accompanies the poleward mass transfer toward the condensing CO₂ polar ice cap, wave cyclones in the winter hemisphere, a strong thermally-direct mean meridional circulation across the equator, with a strong east wind maximum near the equator, and weak east winds over most of the summer hemisphere.

The results of the equinox experiment are like conditions in the earth’s atmosphere. In both hemispheres there are zonal mean west winds in the middle latitudes with wave cyclones in the middle and higher latitudes and east winds in the tropics. In both experiments there are large diurnal tidal components of the circulation.

1. Introduction

Interest in the wind systems on Mars, especially in connection with spacecraft missions to the planet—and recent advances in numerical methods in geophysical fluid dynamics—have motivated and made feasible the simulation of the atmospheric circulation and climate of Mars by numerical integration of the equations of atmospheric motion.

For large-scale atmospheric motion, the hydrostatic approximation can be made and the complete equations of motion then reduce to the so-called “primitive equations”. They are the hydrostatic equation, the horizontal momentum equation, the thermodynamic energy equation, and the continuity equation. These four equations can be cast into the following form:

\[
\begin{align*}
\frac{\partial (\pi)}{\partial t} &= -\text{div}_i(\pi \nu) - \frac{\partial}{\partial \sigma} (\pi \nu) \\
\frac{\partial \nu}{\partial t} &= -\text{div}_i(\pi \nu) - \frac{\partial}{\partial \sigma} (\pi \nu) \\
\frac{\partial \pi}{\partial t} &= -\text{div}_i(\pi \nu) - \frac{\partial}{\partial \sigma} (\pi \nu) \\
\frac{\partial T}{\partial t} &= -\text{div}_i(\pi \nu) - \frac{\partial}{\partial \sigma} (\pi \nu)
\end{align*}
\]

As used here, \( \pi = (P - P_T)/(P_r - P_T) \) is the vertical coordinate, increasing downward, where \( P \) is pressure and \( P_r \) and \( P_T \) are, respectively, the pressures at the lower and upper boundaries of the domain (Phillips, 1967). In addition to \( \pi \), the other independent variables are latitude \( \phi \); longitude \( \lambda \); and time \( t \). The dependent variables are horizontal wind velocity \( \nu \), with eastward component \( u \) and northward component \( v \); temperature \( T \); and \( \pi \) itself as \( (P_r - P_T)/(P_r - P_T) \), where \( P_r \) is a function of \( \phi, \lambda \) and \( t \), and \( P_T \) is a constant. Other quantities are: the geopotential \( \Phi \) of the \( \pi \) coordinate surface; the planetary rotation rate \( \Omega \); the planetary radius \( a \); the gas constant \( R \); the specific heat at constant pressure \( C_p \), and the coefficient, \( \delta = [(P_r - P_T)\pi]^{-1} \). The vertical unit vector is \( \hat{k} \), and \( \text{div}_i \) and \( \text{grad}_i \) are horizontal divergence and gradient operators in the surface of constant \( \sigma \). \( F \) is the horizontal frictional force per unit mass, and \( H \) the rate of heating per unit mass.

To evaluate the right-hand sides of Eqs. (2), (3) and (4), three auxiliary relations are used. These are the
equation for the substantial derivative of \( \sigma \), denoted by \( \dot{\sigma} \), (the "vertical velocity" in \( \phi, \lambda, \sigma \) space),

\[
\tau \dot{\sigma} = - \int_{\sigma}^{\infty} \nabla_n (\nabla \phi) \, d\sigma - \frac{\partial \pi}{\partial t}, \tag{5a}
\]

the pressure tendency equation,

\[
\frac{\partial \pi}{\partial t} = - \int_{\sigma}^{\infty} \nabla_n (\nabla \phi) \, d\sigma - (\tau \dot{\sigma})_{\sigma=1}, \tag{5b}
\]

both of which follow from the continuity equation; and the substantial derivative of the pressure,

\[
\frac{D\pi}{Dt} = \frac{\partial \pi}{\partial t} + \nabla \cdot \nabla_n (\pi), \tag{6}
\]

The value of \( \dot{\sigma} \) at the lower boundary, \( \sigma = 1 \), is identically zero, except in the case of mass transformation at the ground surface when the CO2 condenses or sublimes. This case is discussed in the next section, along with the dependence of the frictional force \( F \) and of the heating \( H \), on the dependent and independent variables. We assume that \( D\pi / Dt = 0 \) at \( P = P_e \), so that \( \dot{\sigma} = 0 \) where \( e = 0 \).

In applying these equations to the atmosphere of Mars, we use the two-level model developed by Mintz and Arakawa. This model has already successfully simulated the gross features of the general circulation of the earth's atmosphere (Mintz, 1965). As shown in Fig. 1, the time-dependent variables carried by the model are the vector wind velocity \( \mathbf{v} \) and the temperature \( T \), evaluated at \( \sigma = \frac{1}{4} \) (level 1) and \( \sigma = \frac{3}{4} \) (level 3), and the surface pressure \( P_T \), (or \( \pi \)). It is assumed, in the model, that the velocity \( \mathbf{v} \) varies linearly with \( \sigma \).

In this model the space differencing scheme is based on the principle developed by Arakawa (1966, 1969). The time-differencing scheme is that of Matsuno (1966a,b) which has the property of damping very high-frequency waves but leaves low-frequency motions practically unaffected.

Our calculation uses a spherical coordinate grid covering the entire planet, with horizontal grid intervals of 9° of longitude around the planet and 7° of latitude from 77S to 77N, plus the two poles. This provides 922 points over the globe. The time step we use, for computational stability, is \((1/240)\)th of a Mars day.

2. Application of the numerical model to Mars

The most important changes in applying the Mintz-Aракава model to Mars, rather than to the earth, are the removal of the oceans and mountains and the change in the mass and composition of the atmosphere.

Based on the results of Kaplan et al. (1964), Kliore et al. (1965), and Spinrad et al. (1966), we assume an atmosphere on Mars whose initial surface pressure is 7.5 mb, made up of CO2 with a partial surface pressure of 5 mb, and N2 with a partial surface pressure of 2.5 mb. With this mass and composition, and the acceleration of gravity, \( g = 3.72 \) m sec\(^{-2} \), the heights of levels 1 and 3 are 13.5 and 3.0 km when the temperature is 200K. We let the heating of the atmosphere consist of the absorption of solar radiation, infrared (IR) radiative transfer, and sub-grid scale convective exchange. Heating of the ground consists of absorption of solar radiation, IR radiative transfer, and convective exchange with the atmosphere. In addition, latent heat release, due to CO2 condensation and sublimation at the ground, is taken into account. Omitted are any effects of water vapor, or of clouds or aerosols, on the heating and cooling.

The rate of heating per unit mass in each atmospheric layer \( i \), resulting from the processes listed above, is

\[
H_i = \frac{\delta}{(\Delta P)_i} (\Delta S_i + \Delta W_i + \Delta C_i), \tag{7}
\]

where \( \Delta S_i, \Delta W_i \) and \( \Delta C_i \) are the differences between
the net energy fluxes at the top and bottom of the layer, arising respectively from solar radiation, IR radiation, and sub-grid scale convection.

The mass of the layer, per unit horizontal area, is represented by \( (\Delta P)_i / g \), where \( (\Delta P)_i \) is the increase in pressure from the top to the bottom of the layer. In this model, the region where the pressure is less than \( P_{L} \) is outside the dynamical domain, but we assume that it absorbs heat. Therefore, for \( i = 1 \) (the upper layer), \( (\Delta P)_1 = (P_1 + P_T) / 2 \); and for \( i = 3 \) (the lower layer), \( (\Delta P)_3 = (P_3 - P_T) / 2 \). Thus, \( H_1 \) is the heating of the entire atmosphere above \( \sigma = \frac{1}{2} \) (the region above \( \sigma = 0 \) as well as the region from \( \sigma = 0 \) to \( \sigma = \frac{1}{2} \)), whereas \( H_3 \) is the heating from \( \sigma = \frac{1}{2} \) to \( \sigma = 1 \). The role of the domain above \( P_T \) is to simulate the IR radiative effect of a stratosphere, the temperature of which is made to depend on the temperatures calculated at levels 1 and 3. The choice of \( P_T \) determines, for given \( T_1 \) and \( T_3 \), the temperature of the region above \( P_T \). We set \( P_T = 0.6223 \) mb, so that with the anticipated values of \( T_1 \) and \( T_3 \) in our experiments the stratosphere temperature would average about 153K, as suggested by the calculations of Prabahakar and Hogan (1965), and Ohring and Mariano (1968).

The heating \( H \) is calculated as follows:

### a. Solar heating

For the heating by solar radiation, we use the method employed by Houghton (1963) for calculating absorption of solar radiation by CO₂ in the earth's stratosphere. When Houghton's near infrared CO₂ absorption band formulae are applied to the pressure and mass conditions in our model, the resulting formulae are:

\[
\Delta S_1 = \left( \frac{r_m}{T} \right) \left( \sin \alpha \right)^4 \times \{465 + (2397 + 531 \ln(\cos \alpha)) \sin \alpha \}^4,
\]

\[
\Delta S_3 = \left( \frac{1}{r_m} \right) \left( \sin \alpha \right)^4 \left( 378 + 657 \sin \alpha \right)^4,
\]

where \( \Delta S_1 \) and \( \Delta S_3 \) are rates of heat input to the upper and lower layers in ergs cm⁻² sec⁻¹, \( r_m / r \) is the ratio of Mars' mean distance from the sun to its actual distance, and \( \alpha \) the solar elevation angle.

### b. Infrared heating

The heating by IR absorption and emission is assumed to be approximately as

\[
W_k(P_k) = K_T \left[ 1 - \tau(T_k) \right] B_r(T_k)
\]

\[
+ \int_{B_r(P_3)} \sum_{i=1}^{n} \left( 1 - \tau_i(P, P_3) \right) dB_i(P),
\]

where \( \tau \) is the mean transmissivity of the band, \( B_r \) the product of bandwidth and blackbody intensity near the band center, \( T_0 \) and \( T_r \) the temperatures, respectively, of the ground and that at the tropopause (where the pressure is \( P_r \)), and \( K_T \) Stefan's constant. The summation in Eq. (10) is over \( n \) spectral intervals in the band, each of which is associated with a transmission function \( \tau_i \) and a blackbody emission \( B_i \). To evaluate Eq. (10), we assume that the temperature varies linearly with height \( Z \) between the tropopause (pressure \( P_T \)) and the top of a very thin surface boundary layer at height \( Z_4 \), as shown in Fig. 1. Within the thin surface boundary layer the air temperature is assumed to vary linearly with height from the ground temperature \( T_0 \) to the air temperature \( T_4 \) \( (Z_4) \), \( T_4 \) being obtained from a linear extrapolation of the temperatures \( T_1 \) and \( T_3 \), computed by the model at levels 1 and 3. The radiative boundary layer is included because there can be large differences between the ground temperature and the air temperature. Above the tropopause (at pressures < \( P_T \)), we assume the temperature to be constant with height and equal to the tropopause temperature \( T_r \), extrapolated from \( T_1 \) and \( T_3 \). With these assumptions, the integral appearing in (10) can be divided into two parts: one from \( P_T \) to \( P_3 \), \( (Z_4) \) and one over the boundary layer from \( P_4 \) to \( P_5 \). We have:

\[
\int_{B_r(P_5)} \sum_{i=1}^{n} \left[ 1 - \tau_i(P, P_3) \right] dB_i(P)
\]

\[
= \frac{R}{g} \int_{(Z_4)}^{P_4} \int_{P_1}^{P_3} \int_{P_1}^{P_3} \left[ 1 - \tau_i(P, P_3) \right] dB_i(P) d\ln P
\]

\[
= \frac{R}{g} \int_{(Z_4)}^{P_4} \int_{P_1}^{P_3} \int_{P_1}^{P_3} \left[ 1 - \tau_i(P, P_3) \right] dB_i(P) d\ln P.
\]

If the boundary layer is very thin, the last term can be written for the surface \( (P_s = P_5) \) in the approximate form,

\[
(T_4 - T_0) \left( \frac{dB_i}{d\ln P} \right) = \left( \frac{RT_4}{Z_4} \right) \int_{P_4}^{P_5} \left[ 1 - \tau_i(P, P_3) \right] d\ln P.
\]

The factor

\[
\left( \frac{RT_4}{Z_4} \right) \int_{P_4}^{P_5} \left[ 1 - \tau_i(P, P_3) \right] d\ln P,
\]

is a function of the boundary layer thickness \( Z_4 \) and is nearly independent of other parameters. We have assumed that \( Z_4 = 5 \) m, which makes this factor approximately equal to 0.04. The contribution of this term to \( W_k(P_s) \) is quite small. On the other hand, for the upward flux at levels far above the ground the boundary layer term in Eq. (11) reduces to

\[
\frac{R}{g} \int_{(Z_4)}^{P_4} \int_{P_1}^{P_3} \int_{P_1}^{P_3} \left[ 1 - \tau_i(P, P_3) \right] dB_i(P) d\ln P
\]

\[
\approx \left[ 1 - \tau_i(P, P_3) \right] \left[ B_r(T_4) - B_r(T_0) \right].
\]
The quantity,

\[ R = \frac{1}{g} \int_{p_T}^{p_T} \left[ \sum_{i=1}^{n} \frac{[1 - \tau_i (P, P_w)] - \ln P_i}{d \ln T} \right] d \ln P, \]

which appears in (11) and, except for the factor \((\Delta Z)^{-1} (T_i - T_a)\), gives the contribution to the net flux from the main part of the troposphere, depends on the temperatures between \(P_0\) and \(P_T\). It was evaluated for five values of \(T_T\), the temperature at \(\sigma = \frac{1}{2}\), in the range 160-280K, for a lapse rate of 3.5K km\(^{-1}\). The resulting points were fitted with a curve that is quadratic in \(T_T\). The necessary transmission functions were evaluated using the formulas of Prabhakara and Hogan (1965) and making use of the Curtis-Godson approximation to account for the pressure-broadening effect. The temperature \(T_T\) was used to evaluate the temperature effect on transmissivity, and the conversion from beam to flux transmissivity was done approximately by multiplying the vertical incidence optical path by the factor 1.67.

The final expressions, in ergs cm\(^{-2}\) sec\(^{-1}\), for the flux differences are:

\[
\Delta W_1 = -1.473 \times 10^{10} Y(T_T) + [1.055 T_T - 1367 - 16580 T_T^{-1}] [T_T - T_a] - 0.1282 \\
\times 10^{10} [Y(T_T) - Y(T_a)],
\]

\[
\Delta W_2 = -0.455 \times 10^{10} Y(T_T) + [1.50 T_T + 171 - 51530 T_T^{-1}] \\
\times [T_T - T_a] - 1.80 \times 10^{10} [Y(T_T) - Y(T_a)] \\
+ 1.302 \times 10^{10} [Y(T_T) / T_T] \\
\times (T_T - T_a) \exp(964.1 / T_T),
\]

where

\[ Y(T_T) = \left[ \exp(964.1 / T_T) - 1 \right]^{-2}, \]

and the temperatures \(T_i\), \(T_a\) and \(T_T\) are derived from \(T_i\) and \(T_T\) by linear interpolation and extrapolation.

c. Convection

Sub-grid scale convective heat transport at the ground, \(C_{si}\), is given in the model by the relation,

\[ C_i = \rho_s C_w \xi \frac{C_P}{T_0 - T_a}, \]

where \(C_P\) is the mean surface air density, \(\xi\) the surface friction velocity, and \(C_w(X)\) a convective heat transfer coefficient which depends on the parameter \(X = [(k \xi \rho_s - T_a) / T_0] \xi / u_k\), where \(k\) is the molecular thermal diffusivity at the surface, and \(T_0\) an approximate global mean surface air temperature, 200K. For stable conditions, \(X < 0\), we assume that \(C_H(X) = C_M\), where \(C_M\) is the momentum drag coefficient for stable conditions. For unstable conditions, \(X > 0\), a partly theoretical, partly empirical formulation for \(C_H(X)\) has been developed by Leovy (1969a). In this formulation, \(C_H(X)\) is given by the implicit relation

\[ C_H(X) = k_0 / \int_0^\infty [(X \psi(\xi) + \xi / \psi(\xi))]^{-1} d\xi, \]

where \(\psi(\xi)\) is an empirical function, \(\xi_t\) an empirical parameter derived from data in the earth's atmosphere, and \(k_0\) is von Kármán's constant. Fig. 2 compares Eq. (16) with the boundary layer heat flux of some terres-
derived from apolation.

\[ \text{Eq. (15)} \]
\[ X = \frac{1}{x} \text{ergs} \text{ cm}^{-2} \text{ sec}^{-1} \]

an empirical atmosphere, omits Eq. 15 some terrestrial observations (Lettau and Davidson, 1957; Vehrenkamp, 1953). The solid curve corresponds to \( \psi(X) = (1 + X)^{1.5} \), as suggested by the observations presented by Dyer (1967). The dashed curve corresponds to the explicit approximation to \( C_H \) used in our Mars calculations, namely:

\[ C_H = 0.142X^{0.14}, \quad \text{if } X \leq 0.153 \]
\[ C_H = 0.204X^{1/3}, \quad \text{if } X > 0.153 \]

Convective heat transfer between levels 1 and 3 is assumed to take place only if the lapse rate determined by \( T_1 \) and \( T_3 \) is sufficiently large. If the lapse rate is unstable (\( \gamma > \gamma_a \)), upward convective heat flux at level \( 2(\sigma = \frac{3}{2}) \) takes place at the rate

\[ C_2 = 4 \times 10^{5} \gamma_a \gamma_a \text{ ergs cm}^{-2} \text{ sec}^{-1} \]

where \( \gamma \) is the actual lapse rate between \( T_1 \) and \( T_3 \), and \( \gamma_a \) is the adiabatic lapse rate. For the assumed composition of the atmosphere, the gas constant is \( R = 0.188 \times 10^{5} \) ergs gm\(^{-1}\) (°K\(^{-1}\)), and the specific heat at constant pressure is \( C_p = 0.879 \times 10^{5} \) ergs gm\(^{-1}\) (°K\(^{-1}\)) in the temperature range 120–300 K; \( \gamma_a = 4.23 \times 10^{-4} (T - 250) \)°K km\(^{-1}\). The constant in (18) was chosen to give a characteristic adiabatic adjustment time of about 0.5 \times 10^4 \text{ sec.}

When the lapse rate is stable, but exceeds the arbitrary value, \( \gamma_a = 2.5 \text{ K km}^{-1} \), and when the surface heat flux \( C_4 \) is directed upward, convective exchange between the layers still takes place in the model. It is assumed that under these conditions some of the heat convected from the surface penetrates to the upper level, and we take

\[ C_2 = C_4(\gamma - \gamma_a)/\gamma_a \gamma_a \]

This formulation allows for the fact that the two-layer representation of the atmosphere will show stability even with an active convective layer which is deeper than the lower model layer, if the atmosphere above the convective layer is stable. Furthermore, even if the convective layer depth averaged over the horizontal region which one grid point represents does not exceed the depth of the lower model layer, we can expect random regions within that area to have deeper convection. Our results show that this model of penetrative convection plays little role in the stable stratosphere and high latitude regions where most of the dynamical activity takes place.

**d. Surface heat balance.**

In order to evaluate radiative and convective fluxes, the ground temperature \( T_0 \) is required. This temperature is obtained from the surface heat-balance equation

\[ (1 - A)S - W_4 - C_4 - D + L = 0 \]

where \( A \) is the surface albedo, \( S \) the downcoming solar radiation at the surface, \( W_4 \) the net upward IR radiative flux at the surface, \( C_4 \) the net upward convective heat flux at the surface, \( D \) the downward conductive heat flux into the soil, and \( L \) the rate of latent heat release due to condensation of CO\(_2\) on the surface. The grid-point albedo values used in these experiments were provided for us by Dr. G. de Vaucouleurs, based upon his photometric map of Mars (de Vaucouleurs, 1967), and are shown in Fig. 3. For the polar regions, the values...
shown in the figure are those estimated to be the minimum value of the albedo for any time of the year.

When there is no solid $\text{CO}_2$ on the surface and no condensation or sublimation, the surface temperature change depends on the soil heat flux $D$. When the major part of the surface temperature change is periodic, as we assume for Mars, $D$ can be expressed with sufficient accuracy in terms of the surface temperature and its time derivative, i.e.,

$$D=\rho \sigma C_\sigma (\kappa_\sigma/2) [0.8(T_a-T_w)+1.2\omega^{-1}(\delta T_a/\delta t)],$$  \hspace{1cm} (21)

where $\rho \sigma$, $C_\sigma$, and $\kappa_\sigma$ are the density, specific heat and thermal diffusivity of the soil, and $\omega$ is the dominant frequency in the temperature variation, in this case the diurnal frequency. The validity of this equation, in the present application, has been demonstrated by Leovy (1966a). In our model, $T_w$, which simulates the temperature at great depth in the soil, is arbitrarily allowed to adjust to $T_a$ with a time constant of 3 days. The thermal-inertia parameter $(\rho C_\sigma \kappa_\sigma)^{1/2}$, appearing in Eq. (21), is assumed to be the same for the entire planet, and is estimated from the radiometric diurnal surface-temperature curve given by Sinton and Strong (1960). The value we use is $8\times 10^4$ cgs units.

The mass of $\text{CO}_2$ condensed on the surface is calculated in the following way. Whenever the surface temperature starts to fall below the frost-point temperature of $\text{CO}_2$ (146.4K at 7.5 mb, an assumed constant in these experiments), or when any condensed $\text{CO}_2$ is already present on the surface, we let $\delta T_a/\delta t=0$, so the surface temperature remains fixed at the $\text{CO}_2$ frost point. Then, neglecting the small contribution from the term in $(T_a-T_w)$, we obtain $D=0$. This allows $L$, and the corresponding rate of $\text{CO}_2$ mass transformation at the surface, to be computed as the residual of the terms in (20). This mass transformation is reflected in the continuity equation by the corresponding mass flux through the surface. Since $(\nu T_a/\nu x)$ at $\sigma=1$ is the flux of mass across the air-ground interface, we have

$$L=(\nu T_a/\nu x)_{x=0},$$ \hspace{1cm} (22)

where $L$ is the latent heat of transformation of $\text{CO}_2$. When $\text{CO}_2$ leaves the atmosphere, by condensation at the surface, energy and momentum also leave the atmosphere. To account for this, we assume that $\text{CO}_2$ condensing on the surface has a temperature corresponding to the linear extrapolation of the potential temperatures at levels 1 and 3, and that its velocity is zero relative to the rotating planet. An additional consequence of $\text{CO}_2$ condensation on the surface is an increase of the surface albedo. We assume that the surface albedo is 0.6 when $\text{CO}_2$ is in solid form on the surface.

### e. Friction

Some of the thermal energy introduced into the system, by the processes described above, is converted to kinetic energy of the wind field. This kinetic energy is dissipated primarily by the frictional force $F$ resulting from the vertical eddy stress $\tau$. For the two-layer model, we can write

$$F_1=\frac{2g}{(P_s-P_T)} (\tau_2-\tau_s),$$  \hspace{1cm} (23)

$$F_2=\frac{2g}{(P_s-P_T)} (\tau_3-\tau_s),$$  \hspace{1cm} (24)

where $(P_s-P_T)/2g$ is the mass per unit area of each layer, $\tau_2$ the stress exerted on the lower layer by turbulent exchange with the upper layer, and $\tau_s$ the stress at the ground. The surface stress is written

$$\tau_s=\rho u_0 s^2 (\nu_1/\nu_s),$$ \hspace{1cm} (25)

where the friction velocity $u_0$ is

$$u_0=C u_1/\nu_s,$$ \hspace{1cm} (26)

and $\nu_s$ is the surface wind obtained by linear extrapolation from the winds at levels 1 and 3. By analogy with the observed condition in the earth's atmosphere (Lettun, 1959), we assume that the momentum drag coefficient $C_u^2$ is not strongly dependent on surface roughness, but is sensitive to the static stability of the surface air. In fact, we ignore any dependence on roughness and assume that $C_u=0.9\times 10^{-5}$ when $T_a\leq T_d$ (stable stratification of the surface layer), and $C_u=3.6\times 10^{-5}$ when $T_a>T_d$ (unstable stratification of the surface layer).

Parameterization of the momentum exchange between atmospheric layers is one of the major uncertainties in the model. It is not yet clear how this should be done for the earth's atmosphere, and the situation for Mars is even less clear. Rather arbitrarily, we have used the following relation for $\tau_2$:

$$\tau_2=\frac{(P_s-P_T)}{2g} C^*(\nu_1/\nu_s),$$ \hspace{1cm} (27)

where $C^*=C_1^*$, if $\gamma<\gamma_m$, and $C^*=C_1^*+(\gamma/\gamma_m)(C_3^*-C_1^*)/(\gamma_m-\gamma_m)$, if $\gamma>\gamma_m$, where $C_1^*=0.2\times 10^{-6}$ sec$^{-1}$ and $C_3^*=4.0\times 10^{-6}$ sec$^{-1}$.

### f. Lateral diffusion

In using this numerical model to simulate the general circulation of the earth's atmosphere (Mintz, 1965), a small lateral diffusion of momentum and heat were used. We did the same for Mars, using the lateral diffusion coefficient $6\times 10^4(\Delta H/300)^4$ [m$^2$ sec$^{-1}$], where $\Delta H$ is the local grid distance in km. It turns out in the experiments that this diffusion transports very little of the atmospheric properties compared with the transports by the mass motions.
3. General description of the numerical experiments

Two numerical experiments were made. The first of these simulates conditions at the time of the southern summer (northern winter) solstice of Mars. The subsolar point was initially set at latitude 24.8°, longitude 0°, with the solar distance ratio, \( r_m/r = 1.197 \).

The second experiment simulates conditions at the southern autumnal equinox of Mars. The subsolar point was initially set at latitude 0°, longitude 0°, with the solar distance ratio, \( r_m/r = 0.929 \).

In both experiments, the solar constant, at mean solar distance \( r_m \) is taken to be \( 0.603 \times 10^9 \) W m\(^{-2}\), which is 0.865 cal cm\(^{-2}\) min\(^{-1}\). Both experiments start from a resting, isothermal atmosphere at 200K, and run for 25 days. The ground temperatures, \( T_g \) and \( T_w \) are also initially set everywhere equal to 200K; and initially there is no condensed \( CO_2 \) on the surface. The absence of initial polar caps does not simulate actual surface conditions on Mars at the solstice and equinox, because the seasonal lags in the true polar caps can represent substantial thermal reservoirs of latent heat. Topographical features are not considered; we have no mountains in our model of Mars.

Some general features of the two experiments are indicated by the time-dependent behavior of the energy of the system. The “mean total kinetic energy” is defined by

\[
K = \frac{a^2}{4gM} \int_{-\pi/2}^{\pi/2} \cos \phi \int_0^{2\pi} (P_s - P_T)(V_1^2 + V_2^2) d\lambda d\phi,
\]

where \( M \) is the total mass of the troposphere, and \( a \) the radius of Mars.

The “mean zonal kinetic energy” is

\[
K = -\frac{1}{4\pi} \int_{-\pi/2}^{\pi/2} (V_1^2 + V_2^2) \cos \phi d\phi,
\]

where the bar operator is defined by

\[
\bar{P}_s = \frac{1}{2\pi} \int_0^{2\pi} P_s d\lambda.
\]

The “mean perturbation kinetic energy” is

\[
K' = K - \bar{K}.
\]

Another quantity of interest is the available potential energy contained in the thermal field (Lorenz, 1953; Dutton and Johnson, 1967). An approximation to this quantity is given by the variance of temperature from its horizontal mean, when it is properly normalized. Thus, an approximation to the “mean total available potential energy” is

\[
A = \frac{a^2}{2gM} \int_{-\pi/2}^{\pi/2} \cos \phi \int_0^{2\pi} (P_s - P_T) \left[ \frac{(T_1 - \bar{T}_1)^2}{Q_1} + \frac{(T_s - \bar{T}_s)^2}{Q_s} \right] d\lambda d\phi,
\]

where

\[
Q_1 = 2\pi \frac{\bar{T}_1(P_s - \bar{P}_s)R_G}{P_s} - \bar{T}_s(P_s - \bar{P}_s)R_G - \bar{T}_s^2, \quad \bar{T} = \frac{\bar{T}_1 + \bar{T}_s}{2},
\]

and \( \bar{T} \) denotes the global mean value for the level.

An approximation to the “mean zonal available potential energy” is

\[
\bar{A} = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \cos \phi \left[ \frac{(T_1 - \bar{T}_1)^2}{Q_1} + \frac{(T_s - \bar{T}_s)^2}{Q_s} \right] d\phi.
\]

Similarly, the “mean perturbation available potential energy” is approximated by

\[
A' = A - \bar{A}.
\]

Fig. 4 shows the course of these quantities during the experiment. In both cases the energies build up rapidly during the first half-day. This initial build-up is due to the thermally driven tides. By the end of the first day, the tidal energy levels off and remains nearly constant, but the mean zonal components of the energies continue to grow as the meridional (south to north) temperature gradients continue to develop. During the first few days the circulations are almost zonally symmetric, with only superimposed tidal perturbations.

Beginning on the fifth day in the solstice experiment, and on the sixth day in the equinox experiment, the perturbation energies grow rapidly, this growth being associated with disturbances which form in the regions of large meridional temperature gradients and large vertical shears of the zonal wind, and is a consequence of the instability of these shear zones. In the solstice experiment, the disturbances occur only in the winter hemisphere, become very intense and produce large energy fluctuations during the remainder of the experiment. As will be seen in the section on synoptic fields, they are the dominant feature of the winter hemisphere. In the equinox experiment, the disturbances are of moderate strength and maintain a comparatively constant energy level in both hemispheres. They are confined to high latitudes. These results are mainly a consequence of the difference in the latitudinal distribution of insolation in the two seasons.

4. Zonal and time average characteristics

Mean values of a number of quantities of interest, obtained by averaging the experimental results over a
circulation: the air rising in the subtropics of the summer hemisphere, moving northward at the upper level, and descending in the winter subtropics. (Rising motion is associated with convergence of the lower level meridional wind and divergence of the upper level wind.) There is some indication of a weaker reverse cell at high latitudes of the winter hemisphere. The upper and lower level meridional winds do not cancel, but there is a net flow toward the pole of the winter hemisphere resulting from the loss of atmospheric mass by CO₂ condensation in the winter polar cap.

In the equinox experiment, both zonal and meridional winds are much weaker. There is westerly flow in both hemispheres, with a tendency for double maxima at the upper level, and only a weak narrow belt of east winds at the equator. The principal cell in the meridional circulation is relatively weak, with rising air in the Southern Hemisphere subtropics and descending air at the equator.

10-day period, days 16 through 25 in each experiment, will be discussed in this section. The results shown are averages of the computer-generated data extracted at half-day intervals, except for the standing waves and diurnal harmonics which are based on data extracted at quarter-day intervals.

a. Wind, temperature and surface pressure

Fig. 5 shows the time and zonal averages of the zonal and meridional winds for the two experiments. At the solstice, the zonal winds are characterized by a strong west wind maximum in the middle latitudes of the winter hemisphere, a fairly strong east wind maximum near the equator, and weak east winds in the middle and high latitudes of the summer hemisphere. The mean meridional circulation shows an intense thermally direct

Fig. 5. Mean zonal and meridional winds. The abscissa is the sine of the latitude in this and subsequent figures.
The solstice winds show intensity variations during the 10-day averaging period. At the upper level, the zonally-averaged west wind component reaches a peak value of 83 m sec\(^{-1}\) at day 18.5, and a minimum of 65 m sec\(^{-1}\) at day 24.5. These fluctuations can also be seen in the oscillations of the zonal kinetic energy, (\(K\)), shown in Fig. 4. In this experiment there is a marked negative correlation between \(K\) and \(K'\), and the alternation of regimes—one having high zonal energy and low perturbation energy, and the other high perturbation energy and low zonal energy—resembles the index cycle variation of the terrestrial atmosphere. This alternation is not found in the Mars equinox experiment.

Fig. 6 shows the time and zonal averages of the temperatures. In the solstice experiment, the entire summer hemisphere and the tropics of the winter hemisphere are characterized by a weak meridional temperature gradient, especially at the upper atmospheric level. The lower level and the ground show larger temperature gradients, with the highest temperatures near the summer hemisphere pole. In the winter hemisphere middle latitudes, both levels and the ground show large temperature gradients. The large vertical shear of the zonal wind, shown in Fig. 5, is geostrophically consistent with this large temperature gradient. In the summer hemisphere at the solstice, the temperature difference between the lower and upper atmospheric levels approaches the adiabatic difference, \(\approx 42\text{K}\). But in the middle and high latitudes of the winter hemisphere the temperature lapse rate is nearly isothermal. In the winter hemisphere the ground temperature is at the frost point of CO\(_2\) from the pole to about latitude 50\(\text{o}\). The mean temperatures of both atmospheric levels are warmer than the CO\(_2\) frost point in all latitudes.

Fig. 6 also shows the corresponding temperatures for the equinox experiment. In all latitudes of both hemispheres the temperature decreases from equator to pole and decreases from the lower to the upper levels. The ground temperature is at the frost point of CO\(_2\) from the poles to about latitude 77\(\text{o}\) in both hemispheres.

The mean zonal surface pressures in the two experiments are shown in Fig. 7, together with the zonal winds extrapolated to the surface from the computed winds at levels 1 and 3. The strong subtropical high and polar low in the winter hemisphere at the solstice contrast sharply with the weaker mean pressure systems at the equinox. The extrapolated surface winds are approximately geostrophic. Because the model does not contain an explicit wind boundary layer, these extrapolated surface winds should be interpreted as occurring at the top of such a boundary layer.
b. The mass budget and the polar caps

The net atmospheric mass transport across latitude $\phi$ by the meridional winds is given by

$$V_{m}^{*}(\phi) = \frac{a \cos \phi}{g} \int_{0}^{2\pi} \int_{P_{T}}^{P} v dP d\lambda.$$  

This quantity, as the average of the twice daily data samples for the two experiments, is shown by the broken-line curves in Fig. 8.

Because the mass change in each latitude is equal to the convergence of the mass transport, $V_{m}^{*}(\phi)$ can also be derived by integration with respect to latitude of the net surface pressure change and net mass loss by condensation in each latitude band. This latter calculation of $V_{m}^{*}(\phi)$ is shown by the solid curve in Fig. 8. The mass transport calculated from the twice-a-day winds is subject to large sampling errors, and for this reason the more accurate mass flux obtained from the mass balance was used to compute the mass flux contributions to the energy and momentum budgets that are discussed later in this section. In the solstice experiment, the total mass of the atmosphere-ice cap system was conserved to better than one part in ten thousand. In the equinox experiment, there was a spurious increase of mass during the 10-day sampling period of 2 parts in a thousand (which is equivalent to a 0.0015 mb surface pressure increase per day over the entire planet.) The reason for this small error is not known, and we distributed the spurious mass increase uniformly over the planet when computing the mass flux in the equinox case.

The very large northward mass flux, in the solstice experiment, is the result of the large rate of mass loss by CO$_2$ condensation in the polar cap. The condensation rate is $4.7 \times 10^{8}$ kg sec$^{-1}$, corresponding to a rate of decrease of the mean surface pressure of 0.011 mb day$^{-1}$. There is also a small net condensation in the equinox case, but as the mass loss rate is only $0.4 \times 10^{8}$ kg sec$^{-1}$, it does not generate a significant poleward mass flow. The polar cap condensation shows that the atmospheric heat transport is not large enough to balance the net radiative cooling in the polar regions. The ground temperatures there fall to the CO$_2$ frost point and, as the CO$_2$ condenses to form and build the polar cap, the latent heat of condensation balances the net energy loss and thermal equilibrium is maintained.

At the winter solstice pole, the cap of solid CO$_2$ covers a large area and, at both levels, the air moving over it cools to about the surface CO$_2$ condensation temperature by the time it reaches the inner region of the cap (Fig. 5). But at the equinox poles, when the caps are small, the air does not remain over the caps long enough to cool to the CO$_2$ condensation temperature.

Fig. 9 shows the maximum and minimum extent of each polar cap during the 10-day averaging periods. In the solstice experiment, the winter hemisphere polar cap reaches to about latitude 90°. In the equinox experiment, each cap extends equatorward to about 70° latitude. The numerical simulations have not included the effects of seasonal storage in the polar caps. Thus, our calculated spring polar cap should not be compared with the observed spring polar cap on Mars. But the storage effect should be relatively unimportant in determining the areal extent of the autumn polar cap and the winter solstice cap, so that these numerical simulations may be compared with observations. The areal extent of our
calculated winter polar cap agrees quite well with the observations reported by Slipher (1962). But, unfortunately, we cannot make any comparison for our autumnal cap, because in that season the true cap seems to be hidden by haze or clouds and its areal extent has not been determined.

Our calculated winter and autumn polar caps correspond closely to the ones derived by Leighton and Murray (1966). This is not surprising, since the most important assumptions pertaining to polar cap formation, surface emissivity near unity (whether ice cap or not) and cloudless skies, were used by Leighton and Murray and ourselves. The main difference in the two calculations lies in the inclusion of atmospheric heat transport in our work. According to our numerical calculations, this transport falls short of that required to prevent condensation in the caps. In the solstice experiment, the radiative loss of the atmosphere and ground poleward of 43.5N is $43 \times 10^{26}$ ergs sec$^{-1}$, but the atmospheric energy transport across 43.5N is only $19 \times 10^{26}$ ergs sec$^{-1}$. The deficit is made up by the latent heat release of CO$_2$ condensation.

We do not believe that inclusion of clouds in the model would change this result. It is true that by providing an enhanced greenhouse effect clouds would slow the rate of condensation on the surface (Leovy, 1960), but to provide an equivalent greenhouse during the period of polar darkness, the temperature of the clouds must be at least as high as the CO$_2$ frost point, and then the radiative loss from the cloud tops would be comparable to the loss from the surface under clear sky conditions. This would lead to further condensation, and presumably to a CO$_2$ precipitation rate comparable to the surface frost deposit rate computed under clear sky conditions (Gierasch and Goody, 1968). The key to the formation of CO$_2$ ice caps on Mars lies in the limited capability of the atmosphere to transport heat into the polar regions, rather than in the local radiative conditions of the polar region.

c. The angular momentum budget.

In the absence of surface topographic features, which can produce pressure torques on the atmosphere, angular momentum on the planetary axis can be transferred between the solid planet and the atmosphere only by the action of the torque exerted by the surface stress, or by the mass transfer due to condensation or sublimation at the surface.

The rates of change of atmospheric angular momentum due to these two effects are shown in Fig. 10. The surface stress exerts a positive torque on the atmosphere where the surface winds are easterly, and a negative torque where the winds are westerly. Thus, in the solstice case, in the winter hemisphere the surface torque transfers westerly angular momentum to the atmosphere in the tropics and subtropics, and extracts westerly angular momentum from the atmosphere in the middle and high latitudes. But there is also a very large extraction of westerly angular momentum from the atmosphere in the tropics of the summer hemisphere, although the mean surface winds are relatively weak there. This is a consequence of the nonlinearity of our surface stress law. The surface westerly winds are strongly developed at the time of day when there is strong surface heating, and they operate then with the larger value of the surface drag coefficient, $C_D = 3.6 \times 10^{-3}$, appropriate to unstable conditions. This may be seen on the synoptic maps shown in the next section.

In the equinox case, westerly angular momentum is transferred to the solid planet in three regions: the subpolar regions of both hemispheres, and in the tropics of the Southern Hemisphere. The latter region is associated with the asymmetric meridional circulation cell and the tropical surface westerlies shown in Figs. 5 and 7.

The change in angular momentum in each latitude belt arises from the surface torque, the convergence of the horizontal angular momentum flux $V_n^\phi$, and the loss of atmospheric angular momentum due to condensation on the surface. The quantity $V_n^\phi(\phi)$ is given by

$$V_n^\phi(\phi) = \frac{a \cos \phi}{g} \int_{\delta}^{\phi} \int_{\Phi}^{\phi} n(\Omega a^2 \cos^2 \phi + au \cos \phi) dx \, d\phi.$$

The two terms within the parentheses correspond to the two angular momentum components: planetary angular momentum, $\Omega a^2 \cos^2 \phi$, and relative angular momentum, $au \cos \phi$. An additional small contribution to the angu-
lar momentum flux comes from the lateral diffusion of momentum, but with the lateral diffusion coefficient we have used this term is small—on the order of 1% of \( V_a^* (\phi) \). The net convergence of angular momentum and its components are shown in Fig. 10. The flux of planetary angular momentum is a consequence of the net mass flux toward the north pole. The convergence of this planetary angular momentum flux accounts for most of the net convergence in middle and high latitudes of the winter hemisphere.

Fig. 10 shows that there is a near balance, in each belt, between net convergence of angular momentum and surface torque. Departures from strict balance are due mainly to sampling error, although there is also a small change in the angular momentum stored in the atmosphere. Departure from strict mass balance, in the equinoctial case, also generates a small error.

The angular momentum flux, \( V_a^* (\phi) \), can be resolved into three components: the flux of planetary angular momentum due to the net meridional mass flux, the flux of relative angular momentum due to the mean meridional circulation, and the flux of relative angular momentum due to the eddy motions (the space and/or time deviations from the zonal and time averaged velocities). These three components are shown in Fig. 11, together with the total flux, \( V_a^* (\phi) \).

Fig. 11 reveals a striking difference in the mechanism for maintaining the high latitude zonal west winds in the two experiments. In the equinocial case, the flux of angular momentum into the two west wind belts is primarily by the poleward flux of relative angular momentum by eddies, just as it is in the terrestrial atmosphere (Lorenz, 1967). In the solstice case, the net poleward flux of angular momentum, which balances the momentum loss due to the surface torque, as well as the smaller loss due to mass condensation in the polar cap, is provided almost entirely by the planetary angular momentum flux. This depends on the net flux of mass, and thus it is the condensation in the winter polar cap which produces the strong westerly winds of the winter hemisphere. The large poleward relative flux by the thermally direct mean meridional circulation in the winter hemisphere is approximately balanced by the equatorward relative flux by eddies.

The eddy flux depends on several classes of eddies. When the velocity fields are averaged over the 10-day period, standing waves along latitude circles are obtained. Other eddy motions are the traveling waves of a fixed period, the most prominent of which is the thermally driven diurnal tide. Harmonic analysis of the standing waves and of the diurnal and higher harmonic tides have been carried out, and are discussed in detail below. The remaining contribution to the eddy flux comes from transient disturbances which do not contribute to the diurnal tide or its harmonics. These transient eddies provide most of the eddy angular momentum flux in both the solstice and equinox experiments, except in the winter hemisphere at the solstice. Examination of the data shows that the large equatorward eddy flux of angular momentum, in the winter hemisphere at the solstice, is produced mainly by the standing waves. The tidal angular momentum flux is of smaller magnitude, and it is toward the equator in both experiments.

d. The heat budget

The heating of the planet as a whole is the difference between the incoming net solar radiation (incoming minus reflected solar radiation) and the outgoing IR radiation. Mean values of these radiative fluxes, averaged for the 10-day periods, in each experiment, are shown in Fig. 12. In the solstice experiment, both incoming and outgoing radiation have maximum values in the polar region of the summer hemisphere. This is a consequence of the fact that the total daily insolation depends on the length of the day. There is an excess of net incoming over outgoing radiation throughout the summer hemisphere, and a deficit in the winter hemisphere, especially over the polar cap. For the equinox experiment, there is near equilibrium, in all latitudes, between net incoming and outgoing radiation, with only small deficits in the extreme polar regions and slight excesses elsewhere. These quantities are not exactly
symmetric about the equator at the equinox, because the surface albedos are not the same in the two hemispheres. Fig. 3 shows lower albedo values in the Southern Hemisphere subtropics which produce the Southern Hemisphere maximum of solar heating. The small differences between net incoming and outgoing radiation are a consequence of the small heat transport capability of the atmosphere.

The heating of the atmosphere alone, by each of the terms in Eq. (7), is shown in Fig. 13 for the two 10-day periods. The direct solar heating is relatively small, and the sub-grid scale convective heating and IR radiative cooling partly balance each other. Nevertheless, during the solstice the atmosphere gains more heat than it loses in most of the summer hemisphere, and loses more heat than it gains in the winter hemisphere. The pattern of atmospheric heating in the equinox case is more complex; the polar regions and Northern Hemisphere tropics lose heat, while the subpolar and middle latitudes, and the Southern Hemisphere tropics, gain heat. This asymmetry between hemispheres, at the equinox, again arises from the albedo asymmetry. It is the cause of the thermally direct meridional circulation between the southern tropics and the northern equatorial region shown in Fig. 5.

The net atmospheric heating at each latitude, as shown in Fig. 13, is not exactly equal to the difference between the net incoming and net outgoing radiation shown in Fig. 12. Over the polar caps, the deficiency is made up by the latent heat of condensation released at the surface. Elsewhere, there is a small heat storage in the ground. There is also a small sampling error, primarily in the outgoing IR radiation which is sensitive to the surface temperature. (A 1K error in surface temperature gives a 2% error in the outgoing radiation.)

When the zonally averaged field of atmospheric temperature is in a steady state, at each latitude the net gain or loss of energy of the atmosphere by radiation and convection will be compensated by the divergence or convergence of the meridional energy flux by the wind. The northward energy flux across latitude $\phi$ is

$$V_T(\phi) = \frac{1}{g} \cos\phi \int_0^{2\pi} \int_{r_p}^{r_s} \psi(C_pT + \Phi + \frac{1}{2}v^2) dP d\lambda.$$  

The three terms on the right correspond to the transport of sensible heat, $C_pT$; potential energy, $\Phi$; and kinetic energy, $\frac{1}{2}v^2$. The last term is of order $(R/C_v)M^2$ relative to $C_pT$, where $C_v$ is specific heat at constant volume, and $M$ the Mach number; the kinetic energy transport therefore can be neglected. An additional meridional energy transfer comes from the sub-grid scale lateral diffusion, but in our model this contributes very little (generally $< 2\%$) to the total energy flux. Thus, the quantity

$$V_{T^*}(\phi) = \frac{1}{g} \cos\phi \int_0^{2\pi} \int_{r_p}^{r_s} \psi(C_pT + \Phi) dP d\lambda$$

is the dominant contribution to energy transport across the latitude circles. The “net convergence” of energy in the atmosphere is the convergence of $V_{T^*}(\phi)$ minus the energy lost when there is a reduction of the mass of
the atmosphere by condensation at the surface \( (C_T T) \) times the rate of mass loss by condensation. This "net convergence" of energy is shown in Fig. 13. It can be seen that there is an approximate but not exact correspondence between the negative of the "net convergence" of \( V_T \) and the net heating by radiation plus convection; the discrepancy again is due to small storage effects and sampling error.

Whereas Fig. 13 showed the "net convergence" of the energy transport, the energy transport, \( V_T \), itself, and its components, the sensible heat and potential energy transports, are shown in Fig. 14. Also shown is the contribution to the sensible heat transport by the mean meridional circulation alone. The contribution to the total potential energy transport by the mean meridional circulation alone is almost indistinguishable from the total potential energy transport. As expected, the transport of energy is away from the heat source regions and toward the heat sink regions, with the mean meridional circulation contributing most of the transport in the tropical and subtropical latitudes of both hemispheres. In these regions, there is a tendency for the sensible heat and potential energy transports to cancel, but with a resultant transport away from the heat source region. This is characteristic of the energy transport by meridional circulations in an atmosphere which is stably stratified.

At high latitudes, in both experiments, the mean meridional circulation contributes very little to \( V_T \), and the energy transport is mainly by the eddies. The total eddy flux of sensible heat is shown in Fig. 15, together with its components due to the standing waves, the diurnal tide and its harmonics, and a residual due to the non-tidal transient eddies.

![Fig. 14. Energy flux \( V_T \) and contributions to it due to the potential energy flux (\( \phi \) flux), and sensible heat flux (\( C_T T \) flux).]

For the standing waves, only wavenumbers 1–3 contribute significantly to the eddy energy flux in the solstice case. Wavenumbers 1 and 2 generally transfer heat toward the winter pole, whereas wavenumber 3 generally transfers heat away from the pole. In the equinox case, the cancellations are almost complete, so that the net flux by the standing eddies is negligible, and is not shown in Fig. 15.

Only the diurnal component contributes significantly to the tidal flux of energy, but this diurnal tidal component accounts for nearly all of the total eddy heat transfer in the tropics and subtropics, in both experiments, and in the middle latitudes of the summer hemisphere in the solstice experiment. It should be noted that this tidal flux depends sensitively on tidal phase relationships which may be badly misrepresented by this model. Elsewhere, the non-tidal transient eddies account for most of the meridional eddy heat flux.

e. The standing waves

An harmonic analysis of the time-averaged fields was made, along each latitude circle, to obtain the amplitudes and phases of the standing planetary waves 1–8, for which temperatures and meridional wind amplitudes are shown in Fig. 16.

The given albedo field, through its effect on the ground temperature, is most likely to be the cause of the standing waves in the two experiments. In the winter hemisphere of the solstice experiment, there is a maximum in the ground temperature and in the atmospheric temperatures and meridional winds at wavenumber 3. The amplitudes of the standing waves are very small at the higher wavenumbers.

In the equinox experiment, there is a maximum in the air temperatures and in the upper level meridional wind, but not in the ground temperature, at wavenumber 2 in
well suited for numerical simulation of the thermally driven tides (Lindzen et al., 1986). Therefore, we do not have much confidence in the amplitudes and phases of the components of the tides produced by the model. Nevertheless, the diurnal tidal mode in the model makes a substantial contribution to the atmospheric transport of heat and we must discuss its behavior.

Fig. 17 shows the variation with latitude, of the amplitude and the phase lag relative to local noon, of the diurnal surface-pressure wave, and of the ground-temperature wave. The diurnal variation of the ground temperature agrees with the observations of Sinton and Strong (1960), because that is how we chose the thermal inertia parameter for Eq. (21); hence, the diurnal variation of ground temperature cannot be used as an independent check of our model.

The surface-pressure variation, shown in Fig. 17, can also be seen by inspection of the 12-hr synoptic surface-pressure maps which are given in the next section. The 12-hr synoptic charts of the level 3 wind field, in the tropics and southern subtropics, at the solstice, also reveals features of the wind field that are confirmed by the harmonic analysis. The low-level diurnal winds sweep southward across the equator near the subsolar point, with speeds of up to 15 m sec⁻¹, and return on the dark side. The upper level meridional winds are nearly

![Fig. 16. Amplitudes of standing waves in the time-averaged temperature and meridional wind fields.](image)

both hemispheres. We believe that the strong maximum in \( T_\theta \) at wavenumber 4, in this experiment, is a spurious consequence of the four-times-daily averaging procedure used to obtain these harmonics. A similar spurious enhancement of \( T_\theta \) wavenumber 4 appears in the solstice experiment at latitudes south of 42°N.

The prominence of wavenumber 3, in the winter hemisphere of the solstice experiment, is related to the three-wave pattern in the albedo. At latitude 42°N, the ground temperature maxima, in wavenumber 3, occur at longitudes 36°, 156° and 276°. Fig. 3 shows that at this latitude there are surface albedo minima (giving maximum absorption of insolation), at longitudes 36°, 170° and 252°. Ridges in the wavenumber 3 lower-level wind field occur at longitudes 45°, 170° and 285° in fair correspondence with the ground temperature maxima. It is not clear why there is no corresponding large enhancement of standing wavenumber 3 at this latitude in the equinox season. The phase of wavenumber 3 in the ground temperature field does correspond to the albedo variations at 42°N (and at 56°N), but the amplitude is small.

**f. Diurnal variations**

Because of its limited vertical resolution and the upper boundary condition, our model is not particularly

![Fig. 17. Amplitudes and phases of surface pressure and ground temperature of the diurnal tide.](image)
Table 1. Energy components.

<table>
<thead>
<tr>
<th></th>
<th>Solstice</th>
<th>Equinox</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total energy input rate (ergs sec⁻¹)</td>
<td>2.03×10¹⁷</td>
<td>1.57×10¹⁷</td>
</tr>
<tr>
<td>Total kinetic energy dissipation rate (ergs sec⁻¹)</td>
<td>5.3×10¹⁰</td>
<td>1.9×10¹⁰</td>
</tr>
<tr>
<td>Fraction dissipated by lateral diffusion</td>
<td>0.29</td>
<td>0.41</td>
</tr>
<tr>
<td>Fraction dissipated by surface stress</td>
<td>0.52</td>
<td>0.31</td>
</tr>
<tr>
<td>Fraction dissipated by stress between layers</td>
<td>0.19</td>
<td>0.28</td>
</tr>
<tr>
<td>Efficiency</td>
<td>2.6×10⁻¹</td>
<td>1.2×10⁻²</td>
</tr>
</tbody>
</table>

180° out of phase with the low-level meridional wind component.

The semi-diurnal and thrice-daily waves are much less intense than the diurnal wave. The semi-diurnal surface-pressure amplitude is only ½ as large as the diurnal amplitude. The semi-diurnal wind amplitude, and the thrice-daily pressure as well as wind amplitudes, are even smaller.

g. Dissipation of kinetic energy

The fractional contributions of the three principal modes of kinetic energy dissipation (lateral diffusion, surface stress, and stress between the two atmospheric layers) to the total rate of dissipation are shown in the third, fourth and fifth lines of Table 1. Some loss of kinetic energy also occurs in the time-differencing scheme, but this loss only affects components whose periods are comparable to one time step (Matsuno, 1966b), and contributes very little to the total dissipation.

The efficiency of the system as a heat engine is the ratio of the total kinetic energy dissipation rate to the total energy input rate, (the incoming minus reflected solar radiation flux). This ratio, shown in the last line of the table, is an order of magnitude smaller than the estimated value for the earth's atmosphere (Eliassen and Kleinschmidt, 1957). The lower efficiency of the atmosphere of Mars is due to the large “windows” in the CO₂ absorption spectrum, the short IR radiative relaxation time, and the fact that a very thin atmo-

sphere cannot transfer much heat. Because of these factors, the temperatures adjust until a very large part of the energy is radiated back to space from the same geographical region in which it is received, and there is consequently very little conversion of available potential energy into kinetic energy.

The kinetic energy dissipation is of the same order of magnitude in the three modes, and the lateral eddy diffusion (which is unimportant for the energy transport) is important for the energy dissipation. The relative importance of the surface-stress dissipation is somewhat greater at the solstice than at the equinox. This is to be expected, because the rate of dissipation by the surface stress increases proportionately to the energy increase to the ½ power, whereas the increase in magnitude of the other two dissipation terms is proportional to the energy increase.

5. Synoptic features

From the time-dependent variables, calculated at every 9° of longitude and 7° of latitude over the planet, we constructed two-daily synoptic charts, or “weather maps”, of Mars. Only a small sample of the maps are shown here.

The maps in Fig. 18 show the initial build-up of the meridional temperature gradients, and the initial development of baroclinically unstable waves, in the solstice and equinox experiments. The maps show the temperatures and vector winds, at level 1, at 3-day intervals following the initial isothermal, motionless, and constant surface-pressure state. The isotherms on the maps (the solid lines) are computer-drawn at intervals of 5K. The vector winds are shown by line segments, with the wind direction toward the dot at each of the points of calculation. The wind speed is proportional to the length of the line segment, scaled according to the strongest wind on each map. The magnitude and location of the maximum and minimum temperatures, and of the maximum wind speed, are given in Table 2.

As indicated earlier, the IR radiative relaxation time is very short in this atmosphere. The net cooling rate is therefore very rapid in those latitudes where there is little or no sunshine; in only a few days the poleward

<table>
<thead>
<tr>
<th>Day</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Longitude</th>
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<tr>
<td>4.0</td>
<td></td>
<td></td>
<td>7.0</td>
<td></td>
<td>10.0</td>
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</tr>
<tr>
<td>Solstice experiment</td>
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<td></td>
</tr>
<tr>
<td>Max T_{l} (°K)</td>
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<td>-35</td>
<td>216</td>
<td>201.4</td>
<td>-35</td>
<td>216</td>
</tr>
<tr>
<td>Min T_{l} (°K)</td>
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<td>90</td>
<td>0</td>
<td>157.1</td>
<td>70</td>
<td>81</td>
</tr>
<tr>
<td>Max</td>
<td>v_{l}</td>
<td>(m sec⁻¹)</td>
<td>50.1</td>
<td>28</td>
<td>189</td>
<td>96.3</td>
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<td>20.0</td>
<td>190.0</td>
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<td>288</td>
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<td>159.1</td>
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<tr>
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<td>(m sec⁻¹)</td>
<td>23.4</td>
<td>63</td>
<td>252</td>
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gradient becomes large and the geostrophic adjustment process builds up the vertical shear of the zonal wind until it becomes baroclinically unstable. In the solstice experiment, there is an initial rapid growth of planetary wave number 4 in the winter hemisphere, as shown in Fig. 18. By the 7th day (as the reader can more readily see by drawing streamlines tangent to the wind vectors in the figures), there are four anticyclonic cells, centered at about latitude 14N. These anticyclones separate weak easterly flow, in the equatorial zone and summer hemisphere, from strong westerlies in the middle latitudes of the winter hemisphere. In the middle and high latitudes of the winter hemisphere, the waves in the temperature field are roughly in phase with the waves in the wind field at level 1. In Fig. 4, we see that the mean perturbation kinetic energy is at its maximum at this time, but the mean zonal kinetic energy is still growing.

In the equinox experiment we see, on the 4th day,
Fig. 19a. Maps from solstice experiment for days 23.5 and 24: top maps, upper level temperatures and winds; center maps, lower level temperatures and winds; bottom maps, surface pressures and surface winds. This arrangement will apply to all other parts of Fig. 19.

only the diurnal tidal motion of the wind field. But, by the 7th day, wavenumber 3 has formed in the Northern Hemisphere. By the 10th day, there are three closed cyclonic cells at level 1 near latitude 70°N, with temperature minima slightly to the west. A comparison with the corresponding maps for days 9 and 11 (not reproduced here), shows that these wave cyclones move eastward with a phase speed of about 8° longitude per day. But although the mean poleward temperature gradients are about the same in the two hemispheres, the disturbances have larger amplitudes in the Northern Hemisphere. This asymmetry of the hemispheres is probably a consequence of the thermal forcing due to the larger third zonal harmonic of the surface albedo in the middle and high latitudes in the Northern Hemisphere.

The fully developed motions are illustrated in Figs. 19 a–d. In each figure the top map shows the tempera-
temperature and winds at level 1; the center map shows the temperature and winds at level 3; and the bottom map shows the surface pressure and the surface wind $\mathbf{v}$, as obtained by linear extrapolation from the computed winds at levels 1 and 3. In these maps the isotherms are computer-drawn at intervals of 10K, and the isobars at intervals of 10 mb. The magnitudes and locations of the maximum and minimum gridpoint values of $T_1$, $T_2$ and $P$, and of the maximum speed of the winds, $|\mathbf{v}_1|$, $|\mathbf{v}_2|$ and $|\mathbf{v}_3|$ are given in Table 3. Again, it is with reference to the maximum wind speed on the individual map that the other wind vectors on that map are scaled.

In the solstice experiment, Figs. 19a and 19b, the changes are so rapid that the maps are given at 12-hr intervals in order to show the continuity of the development. The upper level temperature and wind fields do not exhibit the regularity of the initial development period. The wind field in the Southern Hemisphere is very weak. In the Northern Hemisphere, the largest wave amplitude is in wavenumber 3, in agreement with...
the mean for the 10-day period given in Fig. 16. The disturbances in the high latitudes of the winter hemisphere move from west to east, but one can observe the standing wave component which is related to wavenumber 3 in the surface albedo. In these maps, the wind field does not clearly show the systematic horizontal tilt of the trough and ridge lines. But over the 10-day averaging period, there is a negative correlation between the \( u \) and \( v \) components of the winds, producing the equatorward eddy flux of relative angular momentum shown in Fig. 11. In these maps, which are given at 12-hr intervals, we see in the temperature field at level 3, and in the surface pressure and surface extrapolated winds, the east-to-west progression of the diurnal tides.

In the equinox experiment, Figs. 19c and 19d, the temperature, wind and surface-pressure fields are comparatively symmetrical about the equator. The maps are given at 24-hr intervals, and show the slow eastward
movement of the baroclinic disturbances. The surface low pressure centers move poleward and eastward, and the high pressure centers move equatorward and eastward, with some of the pressure centers dying out and new ones forming during the three days that are shown.

6. Discussion

The properties of the atmosphere of Mars and its boundary conditions which have been taken into account and which are important for understanding the results of these numerical simulation experiments are the following:

1. The small atmospheric mass and high mixing ratio of CO₂ as a radiatively active gas together make the radiative relaxation time so short that the atmosphere quickly adjusts its temperature to the local radiative equilibrium temperature.
2. The underlying surface is dry and has a low heat...
capacity so that it cannot store or transport any heat as does the earth’s ocean.

3) The amount of water vapor contained in the atmosphere is so small that the release of latent heat in water phase transformations is negligible as an atmospheric energy source, in contrast to conditions on the earth. Phase transformations of CO$_2$ can apparently take place, however, and can lead to dynamical effects different from any found in the earth’s atmosphere.

The planetary rotation rate is sufficiently high so that the mean zonal winds must be nearly in geostrophic balance, and the thermal wind relation holds between the vertical shear of the mean meridional wind and the horizontal gradient of zonally averaged temperature. The results show that atmospheric heat transport causes only small modification of the mean temperatures from radiative equilibrium; this is a consequence of the short radiative relaxation time. Even if rather large errors exist in the computed horizontal heat flux, the mean temperature would still approximate the radiative equilibrium distribution. Radiative equilibrium calculations of Prabhakara and Hogan (1965), Carling and Mariano (1968), and Leovy (1966a) indicate that the radiative equilibrium temperature distribution is not sensitive to uncertainties in our knowledge of the CO$_2$ content or surface pressure. We therefore have considerable confidence in the major features of the mean temperatures and in the geostrophically related mean zonal winds as computed with the model. This is supported by interpretations of the Mariner 4 occultation experiment as shown in Fig. 20. The figure compares the computed model temperatures for latitudes and times of day corresponding most closely to Mariner 4 occultation immersion (left) and emersion (right) with temperatures deduced by Kliore et al. (1965) (open circles and error bars), and by Fjeldbo and Eshleman (1967) (shaded bands) based on 100% CO$_2$ model atmospheres.

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**Fig. 20.** Comparison of average model temperature (solid dots and straight lines) for the latitudes and times of day corresponding most closely to Mariner 4 occultation immersion (left) and emersion (right) with temperatures deduced from the occultation experiment by Kliore et al. (1965) (open circles and error bars), and by Fjeldbo and Eshleman (1967) (shaded bands) based on 100% CO$_2$ model atmospheres.

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**Table 3.** Magnitude and location of temperature and surface-pressure maxima and minima and of maximum wind speed.

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<th>Day</th>
<th>21.5</th>
<th>24.5</th>
<th>25.0</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Latitu</td>
<td>Longitu</td>
<td>Longitu</td>
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<tr>
<td>Max $T_1$ ($\text{K}$)</td>
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<td>35.351</td>
<td>208.7</td>
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<td>131.9</td>
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<table>
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<th>19.5</th>
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<tbody>
<tr>
<td></td>
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<td>Longitu</td>
<td>Longitu</td>
</tr>
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<td>49.198</td>
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<td>70.351</td>
<td>63.351</td>
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</table>
unable to transport enough heat poleward to balance the radiative loss. It is possible that our computation is an underestimate of the true poleward heat flux. However, in order to maintain a high enough surface temperature to prevent CO₂ condensation, the atmosphere over the cap must be warmer than our calculations indicate, and the rate of atmospheric energy loss by radiation to space would then be correspondingly greater. There would then be a requirement for the atmosphere to transport even more heat poleward to balance this additional loss. Thus, the actual poleward heat transfer rate would have to be considerably more than twice the amount calculated by the model in order to prevent CO₂ condensation, and that larger heat transport would have to take place with a reduced poleward temperature gradient.

From the dynamical point of view the most interesting consequence of the CO₂ condensation is the maintenance of the strong winter hemisphere west winds by the Coriolis torque due to the net mass flow toward the condensing cap.

If the calculated mean zonal thermal wind is essentially correct, the atmosphere of Mars in middle and high latitudes is unstable with respect to small amplitude baroclinic disturbances in both the winter solstice and equinox cases (Leovy, 1969b). One of the interesting features of the disturbances seen in the solstice experiment is the equatorward transport of angular momentum by the eddies; this is opposite to the normal direction of the eddy angular momentum flux in the earth's atmosphere.

Although the model is not well-suited to represent the phases and vertical structure of the tides, the amplitude of the diurnal surface pressure oscillation is not so sensitive to the limitations in the model (Linzen et al., 1968). The fact that the calculated amplitudes of the diurnal surface pressure and wind oscillations differ from those on the earth is reasonable, in view of the different radiative and convective characteristics of the two planets.

The two numerical simulation experiments for Mars show a large seasonal variation in the character and intensity of the circulation, much more pronounced than the seasonal variation on the earth. This is mainly because there is no large seasonal heat storage such as that provided by the earth's oceans.

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Alfred B. Nelson, of the RAND Corporation, modified the computer program for the Mars experiment and produced the output maps.

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REFERENCES

Klinker, A. et al., 1965: Results of the first direct measurement of occultation experiment: Mars' atmosphere and ionosphere. Science, 149, 1243-1248.